Q.1 (a) A bank charges mark-up @ Re.0.39 per day per Rs.1,000/- . Express rate of mark-up as percent per annum. (01)

(b) Solve the following equation

\[ 2.5x^2 + 3.25x - 16.5 = 0 \] (03)

(c) Find the length of the straight line connecting points A and B having co-ordinates (2, -5) and (-4, 3) respectively. (03)

(d) If 4 : a :: 5 : 8, find “a” (01)

(e) Cost of 3 pen and 8 pencils is Rs 100 whereas cost of 5 pen and 6 pencils is Rs.130. Find the cost of a pen and a pencil. (06)

Q.2 (a) Mr. B is employed as an executive and his present age is 50 years. His present saving is worth Rs.100,000. It is invested in a scheme @ 10% compounded annually. He is due to retire at the age of 60 years. He wishes to receive Rs.80,000/- per annum for 10 years after his retirement. The applicable interest rate then will be 9%. How much amount shall he save and deposit annually at the end of current year and next nine years i.e. upto his retirement to achieve his aim provided he can earn interest of 8% on his annual savings upto his retirement on annual compounding basis. (08)

(b) A note bearing interest of Rs.10,000 at 8% compounded semi-annually for 6 years is discounted, after one year of its issue. Calculate the amount to be received if it is discounted @ 12% compounded annually. (04)

Q.3 (a) Given \( y = U^6 \) and \( U = 3x^4 + 5 \), find \( \frac{dy}{dx} \) (03)

(b) A firm has established the following total revenue (TR) and total cost (TC) functions for its product:

\[
\begin{align*}
TR &= -0.2 Q^2 + 900 Q \\
TC &= 0.03 Q^3 - 0.2 Q^2 + 600
\end{align*}
\]

Where \( Q \) represents the number of items

(i) find the corresponding marginal revenue and marginal cost functions

(ii) use the results of part (i) to find the quantity of the product corresponding to maximum profit. (03)

(c) For the following function, find the second order derivative and evaluate it at \( x = 2 \)

\[ y = (2x + 3) (8 x^2 - 6) \] (03)
Given total cost functions,
\[ TC = 31 + 24Q - 5.5Q^2 + \frac{Q^3}{3}, \]
find the relative minimum or maximum for the function.

Q.4 A factory produces Transformers and Switchgears. The production is carried out in two departments A & B. The machines in department A can work up to 6 hours per day whereas those in department B can work up to 7.5 hours per day. A transformer requires 4 hours in department A & 3 hours in department B, whereas a switch gear requires 2 hours in department A & 5 hours in department B. The profit margin on a transformer is Rs.5,000 and on a switch gear is Rs.6,000. How many transformers and switch gears may be produced to maximize profit in a month having 20 working days. (Use graphical method).

How many machine hours will remain idle during the month in each department at the maximum level of profit.

Q.5(a) A patient’s blood pressure measured daily over several weeks averaged 182 with a standard deviation of 12.6, while that of another patient averaged 124 with a standard deviation of 9.4. Which patient’s blood pressure is relatively more variable?

(b) If the mean monthly salary paid to the three top executives of a firm is Rs.156,000. Can one of them receive a monthly salary of Rs.500,000?

(c) Find the sample variance, sample standard deviation and co-efficient of variation for the following data:

<table>
<thead>
<tr>
<th>Size of orders</th>
<th>No. of orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20 &lt; 30)</td>
<td>3</td>
</tr>
<tr>
<td>(30 &lt; 40)</td>
<td>8</td>
</tr>
<tr>
<td>(40 &lt; 50)</td>
<td>12</td>
</tr>
<tr>
<td>(50 &lt; 60)</td>
<td>6</td>
</tr>
<tr>
<td>(60 &lt; 70)</td>
<td>1</td>
</tr>
</tbody>
</table>

(d) Compute the weighted aggregate price index for the year 2000 on the basis of year 1995 from the following data:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Unit</th>
<th>Average monthly consumption per family</th>
<th>Average price in Rupees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk</td>
<td>Litre</td>
<td>30</td>
<td>25  30</td>
</tr>
<tr>
<td>Flour</td>
<td>Kg</td>
<td>25</td>
<td>10  12</td>
</tr>
<tr>
<td>Cloth</td>
<td>Metre</td>
<td>12</td>
<td>40  60</td>
</tr>
<tr>
<td>Tea</td>
<td>Kg</td>
<td>1</td>
<td>150 200</td>
</tr>
<tr>
<td>Vegetable ghee</td>
<td>Kg</td>
<td>5</td>
<td>50  60</td>
</tr>
<tr>
<td>Eggs</td>
<td>Dozen</td>
<td>4</td>
<td>25  30</td>
</tr>
</tbody>
</table>
Q.6(a) What will be value of Standard Error if all the observed values fall on regression line

(b) While estimating the values of y through a regression line of Y on X, we find that
   at x = 0; y = 8.25
   and at x = 3; y = 12
   Find the value of:
   (i) a (y intercept)  
   (ii) b_{yx} (Regression co-efficient)
   (iii) What does b_{yx} represent

(c) In Q.6 (b) find the value of \( \sum Y \) if \( \sum X = 80 \) and n = 10

(d) Compute co-efficient of co-relation r if b_{yx} = 9.5 and b_{xy} = 0.09

(e) What is the relationship between r_{xy} and r_{yx}

(f) Draw a Scatter Diagram showing negative and linear relationship

Q.7(a) A class contains 10 boys and 20 girls of which half the boys and half the girls have brown eyes. Find the probability that a person selected at random is a boy or has brown eyes.

(b) Of the bolts produced by a factory, 2% are defective. In a shipment of 3600 bolts from the factory, find the expected number of defective bolts and the standard deviation.

(c) A section of a tunnel is lit by 2000 electric bulbs which are kept burning day and night. The manufacturer says that the lives of the bulbs are normally distributed about a mean of 820 hours with standard deviation of 90 hours. How many electric bulbs will be expected to fail before 1000 hours?

(d) Among a department’s 16 trucks, 5 emit excessive amount of smoke. If eight of the trucks are randomly selected for inspection, what is the probability that this sample will include at least 4 of the trucks which emit excessive amount of smoke?

Q.8(a) In estimating population means from sample a 99% assurance is required that sample mean is not different from population mean by more than half of standard deviation. What should be the minimum size of the sample \( Z \propto \frac{\alpha}{2} = 2.58 \)

(b) The test scores of students of Zone A and Zone B yielded the following results

<table>
<thead>
<tr>
<th>Zone</th>
<th>Sample size</th>
<th>Mean Score</th>
<th>Sample Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone A</td>
<td>150</td>
<td>47.3</td>
<td>7.3</td>
</tr>
<tr>
<td>Zone B</td>
<td>100</td>
<td>45.4</td>
<td>6.9</td>
</tr>
</tbody>
</table>

If population scores are normal test the hypothesis that there is no difference between mean scores at a significance level of \( \propto = 0.05 \)

(c) A large normally distributed population has a mean of 1.14 and standard deviation of 0.25. A sample of size 100 is selected. What is the probability that sample mean is greater than 1.16

(THE END)
Q.1(a) A factory produces an average of one defective unit after 10 good units. The cost of producing one unit is Rs.12. The defective unit also costs Rs.12 but is sold as scrap for Rs.2. Determine the selling price of the good units if the manufacturer wants to earn a profit of 20% of total sales.

(b) A company sells its products at 25% above cost. Due to a minor defect in a batch it has to offer discount of Rs.25 per unit. The gross profit percentage after the discount is 10% of discounted sales price. Find the cost of the product.

(c) If $x = 4.762 \times 10^{-7}$
Determine $\log x$

(d) A transport firm is planning to purchase new buses. The budget allocated for this purpose is Rs.50 million. Two types of buses are to be purchased, one costing Rs.300,000 each and the other Rs.500,000 each. If $x$ equals the number of type 1 buses and $y$ the number of type 2 buses,

i) Determine the linear equation which states that the total amount spent on this purchase equals Rs.50 million.

ii) Rewrite the equation in slope-intercept form (take $y$ as dependent variable).

iii) Interpret the meaning of slope and $y$ intercept.

(e) A company earned a profit of Rs.37,000 in its first year of operation. It is estimated that the profit will increase by Rs.7,000 each year. Find the total profit of first 10 years using the method of Arithmetic Progression.

Q.2(a) A person calculates that by depositing Rs.12,500 each year, starting from the end of the first year, he shall be able to accumulate Rs.150,000 at the time of nth deposit if the rate of interest is 4%. Find the number of years in which he can accumulate the required amount.

(b) A food distributor has borrowed Rs.950,000 to buy a warehouse. The loan is for 10 years at an annual interest rate of 12 percent compounded quarterly. What is the amount of quarterly payments which he must make to pay back the loan? How much interest he would pay?

(c) The nominal interest rate on an investment is 12 percent per year. Determine the effective annual interest rate if interest is compounded quarterly.
Q.3 (a) Given a firm’s demand function
\[ Q - 90 + 2p = 0 \]
and its average cost function
\[ AC = Q^2 - 8Q + 57 + 2 \frac{Q}{Q} \]
Find the level of output which
(i) maximizes total revenue
(ii) maximizes total profit

(b) For \( f(x) = 3x^4 - 6x^3 + \frac{5x^2}{2} + 6x \)
Find the point of inflexion.

Q.4 (a) If
\[ A = \begin{bmatrix} 2 & 5 & -7 \\ 1 & 0 & -2 \\ 4 & 8 & 2 \end{bmatrix} \]
\&
\[ B = \begin{bmatrix} -3 & 2 & 5 & 0 \\ 1 & -2 & 3 & -4 \\ -3 & 4 & -2 & 1 \end{bmatrix} \]
Find the product \( AB = P \)

(b) Solve the following equations by using determinants:
\[ 2x_1 + 5x_2 + 3x_3 = 37 \]
\[ 3x_1 + 7x_2 + 2x_3 = 37 \]
\[ 4x_2 + 5x_3 = 43 \]

Q.5 (a) Draw a Pie Diagram to present the following data about a family’s expenditure:
<table>
<thead>
<tr>
<th>Item</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>10,000</td>
</tr>
<tr>
<td>Clothing</td>
<td>2,000</td>
</tr>
<tr>
<td>Education</td>
<td>4,000</td>
</tr>
<tr>
<td>Utilities</td>
<td>4,000</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>5,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>25,000</strong></td>
</tr>
</tbody>
</table>

Note: Exact measurements are not required and the diagram may be drawn freehand

(b) Where first quantile = 37 and 3\(^{rd}\) quantile = 69
Find (i) Quantile Deviation
(ii) Co-efficient of Quantile Deviation

(c) Sales for an industry rose from Rs. 23 million to Rs 27 million, while the industry’s price index rose from 175 to 200. Compute the percentage change in

(i) actual rupees sales;
(ii) sales in constant rupees.
Q.6(a) An organization has collected the following data showing relationship between price charged and quantities sold:

<table>
<thead>
<tr>
<th>Price</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qty</td>
<td>590</td>
<td>560</td>
<td>555</td>
<td>540</td>
<td>525</td>
<td>500</td>
<td>480</td>
<td>475</td>
</tr>
</tbody>
</table>

(i) Determine the regression line equation  
(ii) Compute the maximum price that the company may charge if it wishes to sell only 400 units.  

(b) State in each case whether you would expect a positive correlation, a negative correlation, or no correlation:

i) The ages of husbands and wives;  
ii) The amount of rubber on tires and the number of miles they have been driven;  
iii) Shoe size and IQ;  
iv) The weight of the load of trucks and their petrol consumption.

Q.7(a) A company has 7 directors. In how many ways can the following be selected:

i) Chairman, MD, Executive Director  
ii) 3 members to form a sub-committee

(b) A number between 1 to 10 is selected at random twice. What is the probability that the number selected in the second attempt is greater than the number selected previously.

(c) It is known that 20 percent of all persons given a certain medication get drowsy within two minutes. Find the probabilities that among 14 persons given the medication:

i) at most two will get drowsy within two minutes;  
ii) at least five will get drowsy within two minutes;  
iii) two, three or four will get drowsy within two minutes.

Q.8(a) Calculate standard error of the mean from the following data assuming that population standard deviation is not known:

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>17</td>
</tr>
<tr>
<td>11-20</td>
<td>85</td>
</tr>
<tr>
<td>21-30</td>
<td>129</td>
</tr>
<tr>
<td>31-40</td>
<td>200</td>
</tr>
<tr>
<td>41-50</td>
<td>180</td>
</tr>
<tr>
<td>51-60</td>
<td>120</td>
</tr>
<tr>
<td>61-70</td>
<td>32</td>
</tr>
</tbody>
</table>

(b) To judge certain safety features of a car, an engineer must know whether reaction time of drivers to a given emergency situation has a standard deviation of 0.010 second or whether it is greater than 0.010 second. What can be concluded at 0.05 level of significance if he gets s=0.014 second for a sample size of n=15

(THE END)
Q.1 (a) Rs.1870 is divided into three parts in such a way that half of the first part, one-third of second part, and one sixth of the third part are equal. Find out the value of each part.  

(b) Solve the following by using the rules of logarithm: 

\[ \frac{352.6 \times 0.468 \times 4\sqrt{2.374}}{56.8 \times 0.004362} \]  

(c) Two ingredients, x and y, each measured in kilograms, are used to make two different batches of vitamin C tablets. For batch I, x and y must be mixed in the weight ratio of 5x to 3y. For batch II, the weight ratio is 6x to 2y. Production requirements are 4,000 kgs of batch I and 3,000 kgs of batch II. How much of each ingredient should be purchased to satisfy the production requirements?  

(d) For the profit function \( P = -Q^2 + 17Q - 42 \), find the break even points.  

Q.2 (a) Suppose that Mr Rashid owes Mr Ahmad two sums of money: Rs.100,000 due in 2 years and Rs.60,000 due in 5 years. If he wishes to pay off the total debt now by a single payment, how much he should pay? Assume an interest rate of 8% per annum compounded quarterly.  

(b) An individual plans to borrow Rs.400,000 to buy a new car and would like to keep the payments of Rs.12,500 a month. The loan will be for 3 years at a 12 percent annual rate compounded monthly. Will the monthly payments exceed Rs.12,500 per month limit established by the borrower?  

(c) M/s. ABC Limited is expected to pay Rs.18 every year on a share of its stock. What is the present value of a share if money worth is 9% compounded annually?  

Q.3 (a) Average production cost per unit for a firm is given below: 

\[ AC = 0.02Q^2 - 60Q \]  
where \( Q \) is quantity produced. Find the rate of change of AC at the production point \( Q = 1200 \)  

(b) For the following function, show that the second derivative test fails to characterize the stationary point. 

\[ Y = 2x^3 - 12x^2 + 24x \]  
Characterize the stationary point by means of either the first derivative test or original function test.
(c) \[ Y = 4x^3 - 6x^2 - 9x + 14 \]

Find out the maxima, minima and corresponding values of \( Y \) for the above function.

Q.4 (a) Given:
\[
P = \begin{bmatrix} 7 & -5 & 8 \\ 3 & 5 & -2 \end{bmatrix}
\]
\[
Q = \begin{bmatrix} 3 & 5 & -2 \\ 1 & 4 & -4 \end{bmatrix}
\]

i) Find \(2P + 3Q\)

ii) Find \(4P - Q\)

(b) Determine graphically the solution space (if one exists) for the following:

\[
\begin{align*}
X + Y & \geq 8 \\
2X + Y & \geq 12 \\
X & \leq 10 \\
X & \geq 2 \\
Y & \leq 10
\end{align*}
\]

Q.5 (a) In a perfectly symmetrical frequency curve, the mean of the data is 88. Find median and mode.

(b) In a moderately skewed frequency distribution, mean = 60 and median = 55, Find mode.

(c) For the following consumer price index, calculate the purchasing power of Rupee for each of the year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Consumer Price Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>100</td>
</tr>
<tr>
<td>1997</td>
<td>107</td>
</tr>
<tr>
<td>1998</td>
<td>121</td>
</tr>
<tr>
<td>1999</td>
<td>115</td>
</tr>
<tr>
<td>2000</td>
<td>109</td>
</tr>
<tr>
<td>2001</td>
<td>117</td>
</tr>
</tbody>
</table>

(d) A company sells 3 products A, B and C. From period ‘0’ to ‘1’ prices of A,B and C rose respectively from Rs.80, Rs.25 and Rs.50 to Rs.88, Rs.27 and Rs.53. In period ‘0’, products A,B and C contributed respectively 25, 45 and 30 percent to total rupees sales volume.

(i) Which formula should be used to compute price index for period ‘1’ with period ‘0’ as base?

(ii) Compute the index.
Q.6 (a) Compute two lines of regression from the following data:

\[ r = 0.68, \ x = 68, \ y = 52, \ S_x = 5.12, \ S_y = 5.6 \]

(b) An equal number of families from eight different cities of various sizes were asked how much money they spend on food, clothing and housing per year. The data on city sizes and average family expenditures are given below:

<table>
<thead>
<tr>
<th>City size (000)</th>
<th>30</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>175</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure (Rs.000)</td>
<td>65</td>
<td>77</td>
<td>79</td>
<td>80</td>
<td>82</td>
<td>90</td>
<td>84</td>
<td>81</td>
</tr>
</tbody>
</table>

(i) Compute the correlation co-efficient \( r \).
(ii) Compute the co-efficient of determination and interpret the result

Q.7 (a) A firm, which conducts consumer surveys by mail, has found that 30% of those families receiving a questionnaire will return it. In a survey of 10 families, what is the probability that exactly five families will return the questionnaire?

(b) Records show that the probability is 0.0006 that a car will have a flat tyre while driving through a tunnel. Use the Poisson approximation to the Binomial Distribution to find the probability that at least 2 of 10,000 cars passing through the tunnel will have flat tyres.

(c) The names of 4 men and 6 women are written on slips of paper and placed in a box. Four names are drawn without replacement. What is the probability that 2 are men and 2 are women.

Q.8 (a) A random sample of size 15 is taken from a normally distributed population with mean 60 and the standard deviation 4. Find the probability that the mean of the sample is less than 58.

(b) A random sample of size 6 is drawn without replacement from a finite population of size 16. The standard deviation of the population is 12.

(i) Find standard error of mean
(ii) Find standard error of mean if \( n \) is increased to 10

(THE END)
Q.1 (a) \( y = 3x^2 - 15x + 12 \)

Determine without graphing
(i) Whether the parabola opens downward or upward
(ii) The coordinates of the vertex
(iii) The coordinates associated with the roots of equation

(b) The real estate association in an urban area has determined that a linear function can be used to relate monthly housing sales (the dependent variable, \( y \)) and the average monthly mortgage interest rate (the independent variable, \( x \)). The function is of the form

\[ y = 4500 - 90x \]

(i) Interpret the Y intercept and slope for this function
(ii) If the monthly mortgage interest rate is 14%, what are the expected housing sales for the month?
(iii) If the mortgage interest rate declines from 15% to 12.5%, what is the expected change in housing sales?

(c) A company offers two alternatives for the payment of salary to an executive. Either one may receive Rs.300,000 per month or 1 paisa on the first day of the month, 2 paisas on the second day of the month, 4 paisa on the third day, 8 paisa on the fourth day and so on for the rest of the month i.e. each day getting double of what he has received one day before. Which of the two alternatives should be prefer?

Q.2 (a) A firm’s labour force is growing at the rate of 2 percent per annum. The firm now employee 500 people. How many employees may it be expected to employ in five years’ time?

(b) A non interest-bearing note of Rs.500,000 is due in 4 years from now. If the note is discounted now at 8% compounded semi annually, what will be the proceeds and the compound discount?

(c) A person deposits Rs.30,000 every six months into a retirement account. The account pays an annual interest rate of 12 percent compounded semi-annually. What will be the value of account after 15 years.

Q.3 (a) Differentiate the function,

\[ y = \frac{2}{3x^2} - \frac{x}{3} + \frac{4}{5} + \frac{x+1}{x} \]
(b) Suppose that the demand function for a product is \( p = 400 - 2q \) and the average cost function is \( c = 0.2q + 4 + \frac{400}{q} \) where \( p \) is the price and \( q \) is the number of units.

(i) Determine the level of output at which profit is maximized.
(ii) Determine the price at which maximum profit occurs.
(iii) If the government imposes a tax of Rs.22 per unit, what is the new price for profit maximization?

Q.4  Maximize  
\[ Z = 30x_1 + 24x_2 + 60x_3 \]

Subject to  
\[ 6x_1 + 3x_2 + 5x_3 \leq 30 \]
\[ 2x_1 + 2x_2 + 10x_3 \leq 50 \]
\[ x_1, x_2, x_3 \geq 0 \]

Q.5  (a) In which situation, will Arithmetic Mean, Harmonic Mean and Geometric Mean give the same result?
(b) Given:

Class Interval 0-4 5-9 10-14 15-19 20-24 25-29

Frequency 12 20 48 10 7 3

Required
(i) Calculate Mean Deviation from Median
(ii) Calculate coefficient of Mean Deviation from Median

(c) Calculate the coefficient of dispersion from following set of data:

40 80 70 60 100 160 240 50

Q.6  Construct the cost of living index for the year 2001 on the basis of year 1995 from the following data, using Family Budget Method:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Qty. in 1995</th>
<th>Price per unit of qty. in 1995(Rs.)</th>
<th>Price per unit of qty. in 2001(Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>5000 Kg</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Cloth</td>
<td>1200 Yards</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>Fuel</td>
<td>3000 Gallons</td>
<td>6</td>
<td>8.5</td>
</tr>
<tr>
<td>Misc.</td>
<td>2000 Units</td>
<td>2.5</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Q.7  (a) Calculate the equation of the least squares regression line of \( y \) on \( x \) from the following data:

\[ x \quad 1 \quad 3 \quad 3 \quad 4 \quad 5 \quad 5 \]
\[ y \quad 5 \quad 3 \quad 2 \quad 2 \quad 0 \quad 1 \]
(b) Five students were given following marks in a general knowledge competition by two different judges:

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Ali</th>
<th>Adil</th>
<th>Asif</th>
<th>Ahmed</th>
<th>Ayub</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marks given by Judge A</td>
<td>70</td>
<td>92</td>
<td>80</td>
<td>65</td>
<td>70</td>
</tr>
<tr>
<td>Marks given by Judge B</td>
<td>54</td>
<td>43</td>
<td>43</td>
<td>67</td>
<td>64</td>
</tr>
</tbody>
</table>

**Required:** Calculate Spearman’s Rank Correlation Coefficient.  

Q. 8  
(a) On a single toss of a pair of fair dice, what is the probability that a sum of 7 appears and both dice show a number less than 4?  
(b) A firm of chartered accountants has two vacancies for trainee students and is trying to recruit foundation passed students. In the past, 40 percent of the students who were offered the training contract have not reported to join. If 2 students are offered training contract, what is the probability that at least one will join?  

Q. 9  
(a) Grades on a national aptitude test have been found to be normally distributed with a mean of 250 and a standard deviation of 25. What is the probability that a student selected at random will score between 230 and 256?  
(b) A sample of 16 ten-year old girls had a mean weight of 71.54 pounds and a standard deviation of 12 pounds.  
   (i) What assumption is necessary for the validity of the confidence interval constructed?  
   (ii) What is the estimated standard error of the mean?  
   (iii) Construct a 95% confidence interval for the population mean.  
   (iv) If a 90% confidence interval was to be constructed for population mean from the above data, would the interval be wider or narrower than 95% confidence interval. Explain your answer without actually constructing the interval.  
(c) How large a sample is required to determine the proportion of families in a certain area in need of medical assistance. It is believed that the proportion cannot be greater than 0.35. A 95% confidence interval is desired. The estimate should be within 0.05 of the population proportion. 

(THE END)
Q.1  
(a) Suppose there were 150 traffic accidents last year. What is the number of traffic accidents if the number goes down 20%. What happens if, in the following year, the number of accidents goes up by 20%? (03)

(b) Solve the following exponential function:

\[ (25)^{x+2} = (5)^{3x-4} \]  
(02)

(c) Determine the interest rate needed to have money double itself in 12 years under annual compounding. (02)

(d) If \( f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} \), evaluate the expression \( \frac{x-1}{2x\sqrt{x}} - f'(x) \) (04)

(e) Given \( A = \begin{bmatrix} 6 & -12 \\ -3 & 6 \end{bmatrix} \) \( B = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix} \)

(i) Find \( AB \) (03)

(ii) Why is the product unique

Q.2  
(a) A trust fund for a child’s education is being set up by a single payment so that at the end of 10 years there will be Rs.240,000. If the fund earns at the rate of 8% compounded semi-annually, how much money should be paid into the fund initially?  
(04)

(b) In 6 years a Rs.4 million machine will have salvage value of Rs. 0.4 million. A new machine at that time is expected to sell for Rs.6 million. In order to provide funds for the difference between the replacement cost and the salvage value, a sinking fund is established into which equal payments are made at the end of each quarter. If the funds earn 12% compounded quarterly, how much should each payment be? (06)

Q.3  
(a) Find the derivative of the following function with respect to \( x \).

\[ y = 2x^{3/2} \]  
(02)

(b) The demand for the product of a company varies with the price that the company charges for the product. The firm estimates that annual total revenue \( R \) (in thousand Rupees) is a function of the price \( P \) (in Rupees). Specifically,

\[ R = f(p) = -50p^2 + 500p \]

(i) Determine the price that should be charged in order to maximize total revenue. (03)

(ii) What is the maximum value of annual total revenue? (02)

(c) For the function \( y = x^2 \), find the average rate of change as \( x \) changes from 5 to 10 (03)
Q.4  (a) A firm manufacture two products, \(x_1\) and \(x_2\). For its production, each \(x_1\) requires 2.5 hours in department A, 3 hours in department B and 1 hour in department C. Each \(x_2\) requires 1 hour in department A, 3 hours in department B and 2 hours in department C. The firm can use no more than 20 hours in department A, 30 hours in department B and 16 hours in department C each week. Its profit margin is Rs. 3 per \(x_1\), and Rs 4 per \(x_2\).

(i) Express the data as equations or inequalities
(ii) Graph the inequalities constraints and indicate the feasible region on the graph.

(b) The following function relate annual family income (X) to annual purchases of new clothing (Y):
\[ Y = -600 + 0.06X \]
All measurements are in rupees.
(i) Give a realistic interpretation of the intercept values for this function.
(ii) Interpret the slope co-efficient for this function.
(iii) What are expected new clothing purchases for a family with an annual income of Rs.150,000.
(iv) What is the X intercept for this function. Interpret this point in the context of this problem.

(c) A manufacturer sells a product at Rs. 8 per unit. Fixed cost is Rs.5,000 and the variable cost is Rs.22 /9 per unit. Find the total output at the break-even point.

Q.5  (a) From the following data, determine the average owner occupancy rate (percentage) for the three cities:

<table>
<thead>
<tr>
<th>City</th>
<th>Owner occupancy (percentage)</th>
<th>Number of housing units (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40.3</td>
<td>1,135</td>
</tr>
<tr>
<td>B</td>
<td>56.4</td>
<td>113</td>
</tr>
<tr>
<td>C</td>
<td>62.1</td>
<td>210</td>
</tr>
</tbody>
</table>

(b) The mean temperature in Karachi in the month of January is 16\(^\circ\) C with a standard deviation of 0.5\(^\circ\) C. On January 15, the temperature is 4\(^\circ\) C standard deviation above the mean. What is the temperature on January 15?

(c) Use the following consumer price index to find purchasing power of Rupee for each year relative to the base year and deflate the per capita income:

<table>
<thead>
<tr>
<th>Year</th>
<th>Per Capita Income (Rs.)</th>
<th>Consumer Price Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>6,200</td>
<td>100</td>
</tr>
<tr>
<td>1998</td>
<td>6,700</td>
<td>113</td>
</tr>
<tr>
<td>1999</td>
<td>7,250</td>
<td>120</td>
</tr>
<tr>
<td>2000</td>
<td>7,850</td>
<td>128</td>
</tr>
<tr>
<td>2001</td>
<td>8,600</td>
<td>140</td>
</tr>
<tr>
<td>2002</td>
<td>9,000</td>
<td>150</td>
</tr>
</tbody>
</table>
Q.6 (a) A firm trains employees to use a statistical software package. A random sample of trainees turned in the following performance.

<table>
<thead>
<tr>
<th>Trainee</th>
<th>Hours of training (x)</th>
<th>Number of errors (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

(i) Determine the least square regression line of y on x
(ii) Interpret the co-efficient of regression
(iii) Predict the number of errors for a person with 5 hours of training

(b) The research director of a bank collected 24 observations of mortgage interest rates (x) and number of house sale (y) at each interest rate. The director computed \( \sum x = 276, \sum y = 768, \sum xy = 8690, \sum x^2 = 3,300 \) and \( \sum y^2 = 25,000 \).

Compute the correlation co-efficient between x and y.

Q.7 (a) A sample survey conducted in a city shows that the probabilities are 0.87, 0.36 and 0.29 that a family randomly chosen will own a colour T.V set, a black-and-white T.V. set, or both, respectively. What is the probability that such a family will own at least one of the two kinds of set?

(b) In a graduate school of business an acceptance rate of 30% was reported. If 10 applicants are selected at random, find the probability that:
(i) More than 8 were accepted
(ii) Fewer than 8 were accepted
(iii) Between 5 and 8 (including 5 and 8) were accepted

(c) An analysis of the test scores for an examination revealed that they approximate a normal distribution with a mean of 75 and a standard deviation of 8. The examiner wants to award ‘A’ grade to the upper 10 percent of test grades. What is the dividing point between an A and a lower grade?

Q.8 (a) A population of 500 children has a mean 1Q Score of 100 and a standard deviation of 20 points. If a random sample of 30 children is selected, what is the probability that the mean 1Q of the group exceeds 110?

(b) A population consists of 5 values 6, 8, 10, 12 and 14. How many samples of size 2 are possible, if
(i) sampling is done with replacement
(ii) sampling is done without replacement
(c) Investigating the success of its interviewers, a firm finds that 176 out of 225 interviews attempted by trained interviewers are successfully completed of 310 interviews attempted by untrained interviewers, only 188 are successfully completed. Determine whether these data provide sufficient evidence at the 5% level of significance to indicate a relationship between the training status of interviewers and the outcome of attempted interviews.

(d) The mean monthly income of a random sample of 256 middle level managers of an organization was Rs.45,420 with sample standard deviation of Rs.2,050.

(i) What is the point estimated of mean income of all middle level managers?
(ii) What is the 95% confidence interval of mean income of this group?
(iii) What are the 95% confidence limits?

(THE END)
Q.1 (a) The charges of a hired car are Rs.8 per km for the first 60 km, Rs. 6 per km for the next 60 km and Rs. 4 per km for any further journey. If the balance amount left over with Mr. Rashid is ¼ of what he paid towards the charges of the hired car for traveling 450 km, how much money did he have initially with him? (03)

(b) Solve the following equation by using quadratic formula:
\[ 2 + 6\sqrt{2}y + 9y^2 = 0 \] (04)

Q.2 (a) An individual has purchased Rs.275,000 worth of Defense Savings Certificate. The Certificate expires in 25 years and a simple interest rate is computed quarterly at a rate of 3 percent per quarter. Interest checks are mailed to Certificate holders every 3 months. Determine the interest the individual can expect to earn every three months. What amount can he expect to receive at the end of 25 years? (06)

(b) A person has borrowed Rs.950,000 for a small business. The loan is for five years at an annual interest rate of 8 percent compounded quarterly. What is the amount of quarterly payments to pay back the loan? (06)

Q.3 (a) For the following function, find the second derivative \( \frac{d^2y}{dx^2} \) and evaluate it at \( x = 3 \):
\[ y = \frac{7x^2}{x-1} \] (04)

(b) Average production cost per unit for a company is following:

\[ AC = 0.02Q^2 - 60Q \]

Where average cost per unit \( AC \) is a function of the quantity produced \( Q \):

**Required:**

(i) Find the rate of change of \( AC \) at the following production points:
\[ Q = 1200 \]
\[ Q = 1500 \]
\[ Q = 1800 \] (04)

(ii) What conclusion can you draw about the pattern of costs per unit of production? What are some implications of the result that may be of interest to the company? (02)
Q.4 (a) Use matrix inversion to find the values of $x_1$, $x_2$ and $x_3$ in the following system of linear equations:

\[
\begin{align*}
2x_1 + 4x_2 - 3x_3 &= 12 \\
3x_1 - 5x_2 + 2x_3 &= 13 \\
-x_1 + 3x_2 + 2x_3 &= 17
\end{align*}
\]

(b) Using the Cramer’s Rule or Inverse Matrix Rule, solve the following simultaneous equation:

\[
\begin{align*}
2x + 3y &= 11 \\
x + 2y &= 5
\end{align*}
\]

Q.5 (a) If Rs. 110,000 is to grow to Rs. 250,000 in ten years period, at what annual interest rate must it be invested, given that interest is compounded semi annually?

(b) A team consist of batsmen and bowlers. If $A$ is the number of batsmen and $B$ is the number of bowlers, what will be the inequality constraint that the number of batsmen must be no more than 50% of the total players?

Q.6 (a) The monthly salaries of the 25 employees of XYZ Limited are as follows:

<table>
<thead>
<tr>
<th>Rs.</th>
<th>24,000</th>
<th>9,100</th>
<th>5,000</th>
<th>3,700</th>
<th>14,200</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,600</td>
<td>17,500</td>
<td>6,900</td>
<td>22,900</td>
<td>2,300</td>
<td></td>
</tr>
<tr>
<td>13,900</td>
<td>8,400</td>
<td>9,800</td>
<td>19,300</td>
<td>4,900</td>
<td></td>
</tr>
<tr>
<td>7,800</td>
<td>3,000</td>
<td>4,800</td>
<td>4,200</td>
<td>12,200</td>
<td></td>
</tr>
<tr>
<td>19,000</td>
<td>7,500</td>
<td>4,600</td>
<td>3,000</td>
<td>11,100</td>
<td></td>
</tr>
</tbody>
</table>

The management of XYZ Limited has decided to announce the following bonuses for individuals in the respective salary groups:

<table>
<thead>
<tr>
<th>Group Slab (Rs.)</th>
<th>Bonus (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From 1 to 5,000</td>
<td>3,000</td>
</tr>
<tr>
<td>5,001 to 10,000</td>
<td>6,000</td>
</tr>
<tr>
<td>10,001 to 15,000</td>
<td>9,000</td>
</tr>
<tr>
<td>15,001 to 20,000</td>
<td>12,000</td>
</tr>
<tr>
<td>20,001 to 25,000</td>
<td>15,000</td>
</tr>
</tbody>
</table>

Required:

Compute the average bonus paid by the company to the employees.

(b) Sales of a Product over the last four years have been:

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount in Rs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>100,000</td>
</tr>
<tr>
<td>2001</td>
<td>107,193</td>
</tr>
<tr>
<td>2002</td>
<td>117,384</td>
</tr>
<tr>
<td>2003</td>
<td>119,355</td>
</tr>
</tbody>
</table>
Annual price inflation of the production over the same period has been:
From 2000 to 2001 5.1%
From 2001 to 2002 4.5%
From 2002 to 2003 5.6%

Required
Calculate the index number series of annual sales volume for the production for Year 2000 to Year 2003 (using Year 2000 as the base)

Q.7  (a) The following data is available for sales volume and total costs during a period of four quarters:

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Sales Volume in million</th>
<th>Total costs in million (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>142</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>137</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>149</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>129</td>
</tr>
</tbody>
</table>

Required: By using regression analyses, find out the following:
(i) What is the best estimate of the variable cost per unit (to the nearest Rupee)
(ii) What is the best estimate of the fixed costs per quarter (to the nearest Rupees in million)

(b) For the following two sets of bivariate data, the regression lines for each set are, respectively:
(i) \[ y = 1.94x + 10.83 \text{ (y on x)} \] and \[ x = 0.15y + 6.18 \text{ (x on y)} \]
(ii) \[ y = \text{–1.96x} + 15 \text{ (y on x)} \] and \[ x = \text{–0.45y} + 7.16 \text{ (x on y)} \]

Required:
Find the product moment co-efficient of correlation in each case.

Q.8  (a) The board of directors of a company consists of 12 members, 3 of whom are women. A committee of 3 members is randomly selected from the board.

(i) What is the probability that all members of the committee are men?
(ii) What is the probability that at least one member of the committee is a woman?

(b) The rate of inflation in four successive years in a country was 7 percent, 11 percent, 15 percent and 19 percent. Using the geometric mean, find the average rate of inflation per year.
Q.9 (a) The probability that a computer recovers from a rare virus attack is 0.4. If 15 computers are known to have contracted with this virus, what is the probability that:

(i) at least 12 computers survive;                  \( (03) \)
(ii) from 3 to 8 computers survive.                 \( (03) \)

(b) Mean height, in the training camp, of the players of Pakistan Basketball team is 74.18 inches with a variance of 10.8 inches. If the heights of the players are normally distributed, how many players in a group of 20 would expect to be over 6.5 feet tall? \( (04) \)

Q.10 (a) A distribution company has two branches at Karachi and Lahore. The branches have determined following about the sale to the individual customers:

<table>
<thead>
<tr>
<th>Karachi branch</th>
<th>Lahore Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu^1 = \text{Rs. 11,000} )</td>
<td>( \mu^2 = \text{Rs. 8,500} )</td>
</tr>
<tr>
<td>( \delta^1 = \text{Rs. 1,700} )</td>
<td>( \delta^2 = \text{Rs. 500} )</td>
</tr>
</tbody>
</table>

50 customers from each branch are randomly selected.

**Required**: \( (05) \)

To find out the mean and standard error of \( (x_1 - x_2) \)

(b) (i) Given the following data, test at 0.05 level whether there is any relationship between the level of education and social activities of an individual?

<table>
<thead>
<tr>
<th>Education Level</th>
<th>Social Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Above average</td>
</tr>
<tr>
<td>Graduation</td>
<td>20</td>
</tr>
<tr>
<td>Matriculation</td>
<td>30</td>
</tr>
<tr>
<td>Primary</td>
<td>10</td>
</tr>
</tbody>
</table>

(ii) Interpret your result. \( (07) \)

(THE END)
Module A

Q.1 (a) The population of a city was 8 million on January 1, 2000. The population is growing at the exponential rate of 2 percent per year. What will the population be on January 1, 2005? (03)

(b) A person borrowed Rs.20,000 from a bank at a simple interest rate of 12 percent per annum. In how many years will he owe interest of Rs.3,600. (01)

(c) The number of colour T.V. sets sold by a firm was three times the combined sale of C.D players and radios. If the sales included 72 T.V. sets and 8 radios, how many C.D. players were sold. (02)

(d) Evaluate \( \frac{dy}{dx} \) at \( x = 1 \), if

\[ y = \frac{U}{U + 1} \text{ and } U = 3x^2 - 1 \]  

(04)

(e) Given, \( \begin{bmatrix} 5 \\ 1 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 & 9 & 4 \\ 2 & 1 & 8 \\ 5 & 6 & 1 \end{bmatrix} \)

find, \( AB \) and \( BA \), if possible. (03)

Q.2 (a) The arithmetic mean of profit earned by two companies ‘X’ and ‘Y’ is Rs. 34 million whereas the geometric mean is Rs. 16 million. Find out the profit earned by each company. It is known that company ‘X’ earned more than ‘Y’. (06)

(b) How much money must be invested in an account at the end of each quarter if the objective is to have Rs.225,000 after 10 years. The account can earn an interest rate of 9 percent per year compounded quarterly. How much interest will be earned over the period. (07)

Q.3 (a) Suppose the total cost of manufacturing ‘q’ units of a certain commodity is

\[ c(q) = 3q^2 + q + 48 \]

(i) At what level of production is the average cost per unit the minimum? (08)

(ii) At what level of production is the average cost per unit equal to the marginal cost?
M/s ABC Technologies know that the relationship between their weekly sales $Q$ and weekly profit $PR$ is expressed by the following function:

$$PR = -0.002Q^2 + 10Q - 4000$$

Advise the company about the profit maximizing quantity.

Q.4 (a) Graph the feasibility region for the following system of inequalities

$$2x - y = 5$$
$$3x + y > 0$$
$$x < 4$$

Find all corner points of the feasibility region.

(b) A firm produces three products $X$, $Y$ and $Z$ with a profit of Rs. 20, Rs. 18 and Rs. 16 respectively. Production data are as follows:

<table>
<thead>
<tr>
<th>Machine Hours</th>
<th>Labour Hours</th>
<th>Raw Material Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Y</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Z</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

Availability:
- Machine Hours: 3,000
- Labour Hours: 2,500
- Raw Material Units: 10,000

Set up the initial Simplex Tableau including the necessary slack variables.

Q.5 (a) The following frequency distribution gives the weight of 35 objects, measured to the nearest kg. Draw a histogram to illustrate the data:

<table>
<thead>
<tr>
<th>Weight (Kg)</th>
<th>6-8</th>
<th>9-11</th>
<th>12-17</th>
<th>18-20</th>
<th>21-29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

(b) Following data relates to per unit prices of housing utilities i.e. Electricity, Gas, Water and Telephone:

<table>
<thead>
<tr>
<th>Utilities</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>price</td>
<td>quantity</td>
</tr>
<tr>
<td>Electricity</td>
<td>1.97</td>
<td>62</td>
</tr>
<tr>
<td>Gas</td>
<td>7.90</td>
<td>8.7</td>
</tr>
<tr>
<td>Water</td>
<td>0.29</td>
<td>296</td>
</tr>
<tr>
<td>Telephone</td>
<td>2.40</td>
<td>55</td>
</tr>
</tbody>
</table>

Calculate Fisher’s index for the year 2002, using 2001 = 100. Find the percentage increase in the year 2002 as compared to 2001.
Q.6 (a) The following table shows the data on shelf space allotment \(x\) and sales \(y\):

<table>
<thead>
<tr>
<th>Space allotted (sq. feet-x)</th>
<th>Sales (Number of boxes-y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
</tr>
<tr>
<td>6</td>
<td>38</td>
</tr>
<tr>
<td>8</td>
<td>38</td>
</tr>
<tr>
<td>10</td>
<td>52</td>
</tr>
<tr>
<td>12</td>
<td>54</td>
</tr>
</tbody>
</table>

(i) Determine the equation of least square regression line of \(y\) on \(x\).
(ii) Using the above equation, estimate sales for a space allotment of 7 square feet. 

(b) A computer while calculating the correlation co-efficient between two variables \(X\) and \(Y\) from 25 pairs of observation obtained the following sums:

\[
\begin{align*}
S_X &= 125 \\
S_X^2 &= 650 \\
S_Y &= 100 \\
S_Y^2 &= 460 \\
S_{XY} &= 508
\end{align*}
\]

The following mistakes were discovered at the time of checking:

<table>
<thead>
<tr>
<th>Wrong Values Recorded</th>
<th>Correct Values Need to be Recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X) (Y)</td>
<td>(X) (Y)</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Find out the correct value of the co-efficient of correlation.

Q.7 (a) The following table provides a relative frequency distribution for the size of the farms in the province of Punjab:

<table>
<thead>
<tr>
<th>Size (acres)</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 10</td>
<td>0.087</td>
</tr>
<tr>
<td>10-49</td>
<td>0.192</td>
</tr>
<tr>
<td>50-99</td>
<td>0.156</td>
</tr>
<tr>
<td>100-179</td>
<td>0.173</td>
</tr>
<tr>
<td>180-259</td>
<td>0.098</td>
</tr>
<tr>
<td>260-499</td>
<td>0.143</td>
</tr>
<tr>
<td>500-999</td>
<td>0.085</td>
</tr>
<tr>
<td>1000-1999</td>
<td>0.040</td>
</tr>
<tr>
<td>2000 &amp; over</td>
<td>0.026</td>
</tr>
</tbody>
</table>

A farm is selected at random, determine the probability that the farm selected has:

(i) less than 2000 acres 
(ii) at least 50 acres
(b) In an introductory statistics class, the number of males and females are shown in the following frequency distribution table:

<table>
<thead>
<tr>
<th>Sex</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>17</td>
</tr>
<tr>
<td>Female</td>
<td>23</td>
</tr>
</tbody>
</table>

Two students are selected at random without replacement from the class. Find the probability that the first student selected is female and second is male. (03)

(c) Current medical studies show that 30 percent of the population will suffer from the common cold each winter. A group of 12 people is randomly selected.

i. What is the probability that exactly 5 in the group will have the common cold this winter?

ii. What is the probability that at least 5 in the group will have the common cold this winter?

iii. Compute the mean and variance of the number in the group that will have the common cold this winter. (06)

Q.8 (a) A physician wishes to estimate with 95% confidence the mean ‘Serum Cholesterol’ level of a population. He wishes the estimate to be within 5 units of the true mean. From previous work, he has learnt that the appropriate value of $\sigma = 20$.

How large a sample he should take? (04)

(b) The environmental protection officer of a large industrial plant sought to determine the mean daily amount of sulphur oxide emitted by the plant. A random sample of 10 days’ measurements gave a mean of 9.5 tons per day with standard deviation of 3.24 tons per day. Suppose emissions per day are normally distributed.

Estimate $\mu$ using a 95% confidence interval and interpret your result. (07)

Q.9 For the following distribution, calculate the variance, standard deviation and coefficient of variation:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 - 29</td>
<td>4</td>
</tr>
<tr>
<td>30 – 39</td>
<td>12</td>
</tr>
<tr>
<td>40 – 49</td>
<td>19</td>
</tr>
<tr>
<td>50 – 59</td>
<td>25</td>
</tr>
<tr>
<td>60 – 69</td>
<td>20</td>
</tr>
<tr>
<td>70 – 79</td>
<td>14</td>
</tr>
<tr>
<td>80 - 89</td>
<td>6</td>
</tr>
</tbody>
</table>

(The END)
THE INSTITUTE OF CHARtered ACCOUNTANTS OF PAKISTAN

Foundation Examinations Autumn 2004

September 07, 2004

QUANTITATIVE METHODS

(QMARKS 100)

MODULE A

(Marks 100) (3 hours)

Q.1 (a) A customer paid Rs. 800 for shampoo, hair conditioner and hair spray. The conditioner costs twice as much as the shampoo, and the hair spray costs Rs. 100 less than the shampoo. How much did each item cost? (03)

(b) A clothing store discounts sale price of all its shirts, trousers and jackets by 20%. If \( V_1 \) is the sale value of stock in its 3 branches before the discount, find the value \( V_2 \) after the discount, when

\[
V_1 = \begin{pmatrix}
50,000 & 45,000 & 60,000 \\
100,000 & 120,000 & 75,000 \\
80,000 & 90,000 & 110,000 \\
\end{pmatrix}
\]

(03)

(c) Find the value of \( x \) if

\[5 + 2^{x+2} = 261 \]

(03)

(d) Find the sum to infinity of the following geometric progression.

\[
\frac{1}{1.1} + \frac{1}{(1.1)^2} + \frac{1}{(1.1)^3} + \ldots
\]

(03)

Q.2 (a) Rashid wants to obtain a bank loan. Bank A offers a nominal rate of 14% compounded monthly; Bank B a nominal rate of 14.5% compounded quarterly and bank C offers an effective rate of 14.75%. Which option he should prefer, if all other terms are same. (05)

(b) A home buyer made a down payment of Rs. 200,000 and will make payments of Rs. 75,000 each 6 months, for 15 years. The cost of fund is 10% compounded semi-annually.

(i) What would have been equivalent cash price of the house?

(ii) How much will the buyer actually pay for the house? (06)

Q.3 (a) Find and classify the stationary point of the following function:

\[ Y = \sqrt{x} + \frac{1}{\sqrt{x}} \]

(05)

(b) The cost function for manufacturing a product is given as:

\[ C(x) = 150,000 + 20x - \frac{x^2}{10,000} \]

Find the marginal cost function, and use it to estimate the cost of manufacturing the 50001st unit. (05)

(c) Using Chain Rule, differentiate the following function:

\[ Y = (2 + 3x)^5 \]

(03)
Q.4 An airline wishes to purchase the following aircrafts to meet an estimated demand for 2000 seats:

<table>
<thead>
<tr>
<th>Company</th>
<th>Aircraft</th>
<th>Price of each ($ million)</th>
<th>Seating capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boeing</td>
<td>747</td>
<td>150</td>
<td>400</td>
</tr>
<tr>
<td>Boeing</td>
<td>777</td>
<td>115</td>
<td>300</td>
</tr>
<tr>
<td>Airbus</td>
<td>A 320</td>
<td>60</td>
<td>200</td>
</tr>
</tbody>
</table>

The airline wishes to buy equal number of 747s and 777s. How many should it order to meet the demand for seats within a purchasing budget of $ 710 million. (06)

Q.5 A factory is planning to buy some machines to produce boxes and has a choice of B-1 or B-9 machines. Rs. 9.6 million has been budgeted for the purchase of machines. B-1 machines costing Rs. 0.3 million each, require 25 hours of maintenance and produce 1,500 units a week. B-9 machines costing Rs.0.6 million each, require 10 hours of maintenance and produce 2000 units a week.

Each machine, B-1 or B-9, needs 50 square meters of floor area. Floor area of 1000 square meters and maintenance time of 400 hours are available each week. Since all production can be sold, the factory management wishes to maximize output.

**Required:**

(a) List down the objective function and constraints. (04)

(b) Graph the constraints shading the feasible region. (04)

Q.6 (a) The sales manager of a firm randomly selected 10 sales representatives. He gathered data on the number of calls and the number of units sold by each representative in one month, which is as follows:

<table>
<thead>
<tr>
<th>Sales Representative</th>
<th>Number of calls made</th>
<th>Number of units sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>66</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>28</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>47</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>68</td>
</tr>
</tbody>
</table>

Plot the data in a scatter diagram. Based on the scatter diagram, what observations can you make? (04)

(b) A population consists of 5 values. Its mean, $\mu = 24$ and standard deviation, $\sigma = 2.83$

(i) How many samples of size 2 are possible, if sampling is done with replacement? Without replacement?

(ii) Based on all possible samples of size 2, a sampling distribution of means is formed. Find its mean ($\mu_{\bar{x}}$) and standard error ($\sigma_{\bar{x}}$) (04)
Q.7 (a) A farmer wishes to predict the number of tons per acre of crop which will result from a given number of applications of fertilizer. Data collected is given below:

<table>
<thead>
<tr>
<th>Fertilizer applications</th>
<th>1  2  4  5  6  8  10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tons of crop per acre</td>
<td>2  3  4  7  12  10  7</td>
</tr>
</tbody>
</table>

Find a suitable linear regression relationship to help the farmer in making the required prediction and from your result predict the number of tons per acre of crop from 7 applications of fertilizer. (07)

(b) The weights of a group of children are approximately normally distributed with mean = 15.0 kg and standard deviation = 1.75 kg.

(i) What proportion of children will weigh between 13 and 16 kg?
(ii) What proportion of the children will weigh 13 kg or more?
(iii) The heaviest 5% of the children are to be studied further. What is the cut off point between those who will be studied, and those who will not be studied? (07)

Q.8 (a) The Board of Directors of a company consists of 8 men and 4 women. A 4 number committee is to be chosen at random from the Board:

(i) What is the probability that all 4 members of the committee will be women?
(ii) What is the probability that all 4 members of the committee will be men?
(iii) Why the sum of the probabilities for (a) and (b) does not equal 1? Explain. (05)

(b) It is estimated that 0.5% of the callers to a department will receive a busy signal. Using the ‘Poisson Distribution’, find the probability that of 1200 callers at least 5 will receive a busy signal. (05)

Q.9 The personnel manager of a firm is concerned about ‘absenteeism’. He decided to sample the records to determine if absenteeism is distributed evenly throughout the six day week. The sample results are:

<table>
<thead>
<tr>
<th>Weekday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>12</td>
<td>9</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Absent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) What is the expected frequency for each day? (01)
(b) How many degrees of freedom are there? (01)
(c) What is the chi-square value at 1% level of significance? (01)
(d) Using the chi-square test of significance, compute $\chi^2$. (02)
(e) Is the null hypothesis rejected at 1% level of significance? (01)
Q.10 Calculate 1st quartile, 5th decile and 70th percentile for the following frequency distribution:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-5</td>
<td>6</td>
</tr>
<tr>
<td>6-8</td>
<td>8</td>
</tr>
<tr>
<td>9-11</td>
<td>9</td>
</tr>
<tr>
<td>12-14</td>
<td>14</td>
</tr>
<tr>
<td>15-17</td>
<td>16</td>
</tr>
<tr>
<td>18-20</td>
<td>17</td>
</tr>
<tr>
<td>21-23</td>
<td>11</td>
</tr>
<tr>
<td>24-26</td>
<td>8</td>
</tr>
<tr>
<td>27-29</td>
<td>7</td>
</tr>
<tr>
<td>30-32</td>
<td>4</td>
</tr>
</tbody>
</table>

Q.11 Use the following consumer price indices to find purchasing power of a rupee for each year relative to the base year and deflate the per capita income.

<table>
<thead>
<tr>
<th>Year</th>
<th>Per capita income Rs.</th>
<th>Consumer price index</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>19,500</td>
<td>100</td>
</tr>
<tr>
<td>2001</td>
<td>21,200</td>
<td>113</td>
</tr>
<tr>
<td>2002</td>
<td>22,300</td>
<td>120</td>
</tr>
<tr>
<td>2003</td>
<td>24,100</td>
<td>128</td>
</tr>
</tbody>
</table>
Q.1 (a) An investor receives a total of Rs.5,700 per annum in interest from 3 stocks yielding 4%, 5% and 8% per annum respectively. The amount at 4% is 20,000 more than the amount invested at 5%, and the interest from the 8% investment is 8 times the interest from the 5% investment. How much money is invested in each stock? (04)

(b) Solve the following, using logarithm:
\[ \frac{1}{3^x} = (12)^{x+2} \] (03)

Q.2 (a) An auditorium has 20 seats in the front row, 25 seats in the second row, 30 seats in the third row and so on for 13 rows. How many seats are there in the thirteenth row. (03)

(b) A borrower signs a note and agrees to pay Rs.10,000 in 9 months at 10% simple discount. How much does this borrower receive? (03)

Q.3 (a) Mr Rashid invested Rs.60,000 in a company but found that his investment was losing 6% of its value per annum. After two years, he decided to pull out what was left of the investment and place at 4% interest compounded twice a year. Will he recover his original investment after 6 more years? (06)

(b) A firm wants to deposit enough in an account to provide for insurance payments over the next 5 years. Payments of Rs.27,500 must be made each quarter. The account yields an 8% annual rate compounded quarterly. How much be deposited to pay all the insurance payments? (06)

Q.4 (a) Differentiate the following function:
\[ y = \left(3x + e^x\right)^3 \] (03)

(b) Given,
\[ f(x) = \left(\frac{3x + 2}{4x^2 - 5}\right)^5 \text{, find } \frac{dy}{dx} \] (04)

Q.5 The cost of producing \( x \) units of a product is
\[ C = 20 \cdot x^{\frac{3}{2}} + 300 \]

Required:
(i) Estimate the marginal cost, when \( x = 100 \), and when \( x = 1000 \)
(ii) Compare the actual marginal cost when \( x = 100 \) with the approximation found in (i)
(iii) Find the average cost when \( x = 100 \) and when \( x = 1000 \). (06)
Q.6 (a) The following two matrices are equal. Find the values of p, q, r and n,

\[ A = \begin{bmatrix} p & q \\ 1 & r \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 \\ n & 0 \end{bmatrix} \]

(b) Suppose a factory has received a Purchase Order for three products as follows:

\[ p_1 = 7 \text{ units}; \quad p_2 = 12 \text{ units}; \quad p_3 = 5 \text{ units} \]

This order is represented by a row vector:

\[ P = [7 \ 12 \ 5] \]

Raw material of 4 kinds, M1, M2, M3, M4 required to produce each product is represented by the matrix:

\[ Q = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \begin{bmatrix} M1 & M2 & M3 & M4 \\ 2 & 3 & 1 & 12 \\ 7 & 9 & 5 & 20 \\ 8 & 12 & 6 & 15 \end{bmatrix} \]

The cost for each material is represented by:

\[ C = \begin{bmatrix} 10 \\ 12 \\ 15 \\ 20 \end{bmatrix} \]

Compute each of the following and interpret each one:

(i) PQ (ii) QC (iii) PQC.

Show your computation.

Q.7 Prices for four items for 1995 and 2004 are given in the following table:

<table>
<thead>
<tr>
<th>Item</th>
<th>1995</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price (Rs)</td>
<td>Quantity sold</td>
</tr>
<tr>
<td>A</td>
<td>6</td>
<td>2000</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>100</td>
</tr>
</tbody>
</table>

(i) Using the ‘Laspeyres’ formula, calculate a weighted index of price for 2004 (1995=100). Interpret your result.

(ii) Calculate an index of value for 2004 (1995=100). Interpret your result.
Q.8 (a) The age and price data for a sample of 11 Nissan Sunny Cars are presented in the following table:

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Price (in Rs. ‘000’)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>85</td>
</tr>
<tr>
<td>4</td>
<td>103</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>82</td>
</tr>
<tr>
<td>5</td>
<td>89</td>
</tr>
<tr>
<td>5</td>
<td>98</td>
</tr>
<tr>
<td>6</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>169</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
</tr>
<tr>
<td>7</td>
<td>48</td>
</tr>
</tbody>
</table>

(i) Calculate the linear correlation coefficient, r, of the data.

(ii) Interpret the value of r obtained in part (a) in terms of linear relationship between the two variables.

(b) If 8 members of a tennis club are classified A players, 6 are classified B players and 10 are classified C players, in how many different ways can 2 players from each group be chosen to represent the club.

Q.9 (a) Out of 12 eggs in a refrigerator, 2 are rotten. From these 12 eggs, 4 egg are selected at random to make a cake. What are the probabilities that

(i) Exactly one is rotten.

(ii) At least one is rotten.

(b) Suppose a poll of 10 employees is taken in a large company. The purpose is to determine x, the number who favour unionization. Suppose that 40% of all company’s employees favour unionization:

(i) Find the mean and standard deviation of x

(ii) Find the probability that x < 4

(iii) Find the probability that x > 6

(c) Suppose a population consists of 15 items, 10 of which are acceptable. A sample of 4 items is selected without replacement. What is the probability that exactly 3 are acceptable?

Q.10 (a) If we take a sample from an infinite population, what will happen to the standard error of the mean when the sample size is increased from 60 to 240.
A firm believes that the mean cost to process a sales order is Rs. 132.50. The cost controller fears that the average cost of processing is more than that. A random sample of 100 orders has a sample mean of Rs. 133.5 and standard deviation of Rs. 5. Level of significance = 0.05

Required:

(i) State the Null Hypothesis and Alternative Hypothesis.

(ii) Which test distribution (Z or t) will be used and why?

(iii) Whether the ‘Ho’ will be rejected or not; give reason for your conclusion.

(iv) Find the p– value and interpret your result.

Q.11 A certain change in a process for manufacturing of component parts is being considered. Samples are taken using both the existing and the new procedures so as to determine if the new process results in an improvement. If 75 of 1500 items from the existing procedures are found to be defective and 80 of 2000 items from the new procedures were found to be defective, find a 90% confidence interval for the true difference in the fraction of defectives between existing and the new process.
Q.1 Determine which pairs of the given lines are parallel, perpendicular or neither. Give reason in each case and show your calculations:

(a) \[3x = y + 7 \quad \text{and} \quad x + 3y = 7\] (02)
(b) \[x + y = 4 \quad \text{and} \quad x - 2y = 3\] (02)
(c) \[2x + 3y = 5 \quad \text{and} \quad 4x + 5 = -6y\] (02)

Q.2 (a) Solve the following equation:
\[4m^4 - 9m^2 + 2 = 0\] (04)
(b) The sum of 16 terms of an Arithmetic Progression, whose last term is 250, is 2800. Find the first term and common difference. (04)

Q.3 (a) Bashir owes Rs.50,000 to Arshad due to a court decision. The money must be paid in 10 months with no interest. Suppose Bashir wishes to pay the money now. What amount should Arshad be willing to accept? Assume simple interest of 8% per annum. (03)

(b) Farhan borrowed Rs.100,000 for one year at 12% annual interest compounded monthly. The loan is to be paid in equal monthly installments.

(i) Determine the amount of each installment. (02)
(ii) Calculate principal repayment included in first installment. (02)
(iii) Find the total interest paid during the year. (02)

Q.4 (a) The demand function for a certain product is given by
\[p = \frac{50,000 - x}{25,000}\]
where \(p\) is the price in rupees and \(x\) is the demand
Find the marginal revenue when \(x = 10,000\) units. (04)

(b) A small company must hire expensive temporary staff to supplement its full-time staff. It estimates that the monthly costs, \(c(x)\), are related to the number of full time employees by the function:
\[c(x) = 250x + \frac{16,000}{x} + 100\]
where \(x\) is the number of full time employees
How many full-time employees should the company have, to minimize the costs? (07)
Q.5  (a) An investment group is planning to establish shopping centres in 5 markets. They have estimated the initial cost and the rent (both in rupees per square foot) for each centre, respectively, as follows:

<table>
<thead>
<tr>
<th>Initial cost (in rupees)</th>
<th>180</th>
<th>100</th>
<th>80</th>
<th>100</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent (in rupees)</td>
<td>27</td>
<td>15</td>
<td>10</td>
<td>20</td>
<td>17</td>
</tr>
</tbody>
</table>

Write this information first as a 5 x 2 matrix and then as a 2 x 5 matrix.  

(b) Show that $B^2 = B$, where

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.6  (a) Sketch the feasible region for the following set of constraints:

\begin{align*}
3y - 2x & \geq 0 \\
y + 8x & \leq 52 \\
y - 2x & \leq 2 \\
x & \geq 3
\end{align*}

(b) Find the maximum value of the objective function $Z = 5x + 2y$, by determining the corner points of the feasibility region sketched in Q.6 (a).  

Q.7  (a) A student received a grade of 79 in a final examination in mathematics for which the mean grade was 76 and the standard deviation was 10. In the final examination in economics, for which the mean grade was 91 and the standard deviation was 5, she received a grade of 92.

In which subject, her relative standing was higher.  

(b) A firm manufactures refrigerators at three plant locations A, B and C. For each location, the firm maintains a record of the refrigerators made at that plant which require repair before expiration of warranty. Following is the summary of the information about repairs:

- Plant ‘A’ produces 30% of all refrigerators and has a repair rate before warranty expiration, of 5%.
- Plant ‘B’ produces 45% of all refrigerators and has a repair rate before warranty expiration, of 3%.
- Plant ‘C’ produces 25% of all refrigerators and has a repair rate before warranty expiration, of 7%.

If a refrigerator requires repair before warranty expiration, what is the probability that it was manufactured at Plant ‘B’?
Q.8 In an organization of 30 persons, sickness and absence records were kept daily for 3 months. The number of workers absent each day are shown below. Find the mean, variance and standard deviation of the number of employees absent per day.

<table>
<thead>
<tr>
<th>No. of employees absent (x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of days (f)</td>
<td>44</td>
<td>19</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Q.9 A study by the transportation department on the effect of bus-ticket prices upon the number of passengers, produced the following data:

<table>
<thead>
<tr>
<th>Ticket price in Rs. (x)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Passengers (y)</td>
<td>800</td>
<td>780</td>
<td>780</td>
<td>660</td>
<td>640</td>
<td>600</td>
<td>620</td>
<td>620</td>
</tr>
</tbody>
</table>

(a) Develop the least square regression line of y on x
(b) How do you interpret the slope of the above regression line?
(c) Estimate the number of passengers if the ticket price is 8.5 rupees.

Q.10 The following table contains the data on the heights and weights of a group of women:

<table>
<thead>
<tr>
<th>Height (x in inches)</th>
<th>Weight (y in pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>140</td>
</tr>
<tr>
<td>69</td>
<td>150</td>
</tr>
<tr>
<td>62</td>
<td>110</td>
</tr>
<tr>
<td>71</td>
<td>170</td>
</tr>
<tr>
<td>66</td>
<td>120</td>
</tr>
<tr>
<td>68</td>
<td>150</td>
</tr>
<tr>
<td>70</td>
<td>150</td>
</tr>
<tr>
<td>67</td>
<td>130</td>
</tr>
<tr>
<td>63</td>
<td>120</td>
</tr>
<tr>
<td>65</td>
<td>100</td>
</tr>
</tbody>
</table>

Compute the co-efficient of correlation and co-efficient of determination and interpret your results.

Q.11 (a) Find the values of –Z and Z if the standard normal curve area between –Z and Z is 0.9700.
(b) A normal distribution has a mean = 61.6. Find its standard deviation if 20% of the total area under the curve lies to the right of x = 70.
(c) In an experiment to determine the amount of time required to assemble a toy, the assembly time was found to be a random variable having approximately a normal distribution with \( \mu = 27.8 \) minutes and \( \sigma = 4.0 \) minutes. What is the probability that this kind of toy can be assembled in less than 25.0 minutes?
Q.12 (a) The length of babies at birth has a mean, \( \mu = 19 \) inches and a standard deviation, \( \sigma = 1.3 \) inches. Samples of 100 babies are chosen:

(i) Find the mean and standard deviation of the sampling distribution of means. (02)

(ii) What is the probability that one of the samples of 100 babies will have a mean less than 18.7 inches? (03)

Q.12 (b) A survey is being planned to determine the mean amount of time, senior citizens (older than 60 years) watch T.V. A pilot survey indicated that the mean time per week is 12 hours with a standard deviation of 3 hours. It is desired to estimate the mean viewing time within a quarter hour. 95% confidence level is to be used. How many senior citizens should be surveyed? (03)

Q.13 A random sample of the records of 300 students of a college shows that 213 of them have G.P.A greater than 2.5. Find a 90% confidence interval for the proportion of students of this college who have G.P.A greater than 2.5. Interpret your result. (05)

Q.14 I.Q of students at a college are normally distributed with a mean = 112 and a standard deviation = 12. Professor Hameed believes that I.Q of students in his Statistics class do not have a mean I.Q of 112. To prove this point, a random sample of 5 students from Statistics class is chosen. Their I.Q Scores were found to be 115, 124, 131, 129 and 120. Test the hypothesis that the mean I.Q of the students of the class is the same as that of the population of students at the college. Use \( \alpha = 0.05 \). (07)

(THE END)
Q.1 (a) A shopkeeper sells all his items at 40% above costs. If he allows a special discount of 15% to his friend on selling price, compute his profit as a percentage of cost.

(b) Solve the following equation by using the laws of logarithms:

\[
\frac{(564.2)(8.94)^{2/3}}{(0.4577)(482.7)}
\]

Q.2 (a) A company manufactures two products, A and B. Each unit of A takes 20 hours in assembling department and 5 hours in finishing department. Each unit of B takes 10 hours for assembling and 3 hours for finishing. If the total number of hours available in assembling and finishing departments are 450 and 130 respectively, calculate the number of each product that can be produced provided both departments are fully utilised.

(b) A man got a lottery of Rs.7,000,000 on 15\textsuperscript{th} February. He paid income tax of 25% on this amount. From the remaining amount he decided to donate one paisa on the first day, 2 paisas on second day, 4 paisas on third day and so on, commencing from March 1, till the end of March. Determine whether the amount of money available with him is enough to pay these donations?

Q.3 (a) An investor can earn 9.1% interest compounded semi-annually or 9% interest compounded monthly. Determine which option he should prefer.

(b) A company is planning to pay off a loan at the end of the third year. The amount then payable would be Rs. 120 million. The company wants to create a sinking fund for this purpose. The company will earn a 10% annual rate compounded quarterly on all deposits. Contributions to the fund are to be made at the end of each quarter. What is the required quarterly deposit?
A construction company is considering a project which would cost Rs. 1,700,000 now plus Rs. 800,000 at the end of year 1. This investment would result in net earnings of Rs. 500,000 per annum from year 2 to year 8, to be received at the end of the year. The company requires a return of 11% per annum on its investments.

(i) Calculate the Net Present Value of the project. 
(ii) Determine whether the project is viable?

Q.4 (a) The manufacturing cost of a product consists of fixed overheads of Rs. 100,000, material costs of Rs. 2 per unit and labour cost equal to \( \frac{x^2}{90} \) for \( x \) items produced.

At what level of production, the average cost per unit will be minimum?

(b) Differentiate the following function:
\[ y = \left( x + \frac{1}{x} \right)^2 \]

Q. 5 (a) Find the matrices \( X \) and \( Y \), if:
\[ X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \quad \text{and} \quad X - Y = \begin{bmatrix} 3 & 9 \\ 0 & -1 \end{bmatrix} \]

(b) Draw the graph of the following linear inequalities and shade the common area:
\[
\begin{align*}
5x + 4y & \leq 200 \\
3x + 5y & \leq 150 \\
5x + 4y & \geq 100, \quad x \geq 0, y \geq 0
\end{align*}
\]

(c) A firm produces three products A, B and C with a contribution of Rs. 200, 180 and 160 respectively. Production data are as follows:

<table>
<thead>
<tr>
<th>Products</th>
<th>Machines Hours</th>
<th>Labour Hours</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Availability</td>
<td>3,000</td>
<td>2,500</td>
<td>7,000</td>
</tr>
</tbody>
</table>

You are required to set up the initial Simplex Tableau including the necessary slack variables.
Q.6 The following table shows the distribution of monthly salaries of a company’s employees:

<table>
<thead>
<tr>
<th>Salary Scale</th>
<th>3000-3500</th>
<th>3501-4000</th>
<th>4001-4500</th>
<th>4501-5000</th>
<th>5001-5500</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of employees</td>
<td>300</td>
<td>270</td>
<td>210</td>
<td>170</td>
<td>100</td>
</tr>
</tbody>
</table>

You are required to calculate:
(a) Mean deviation from Median
(b) Co-efficient of Mean deviation from Median. (07)

Q.7 (a) For the following data set, construct a scatter diagram and comment on the relationship between $x$ and $y$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>12</td>
<td>7</td>
<td>6</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

(03)

(b) Calculate the co-efficient of correlation ‘$r$’ for the above data set. Does it seem consistent with the above scatter diagram? (06)

Q.8 Data regarding age and price for a sample of 11 cars of a particular type are given below:

<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>Price (million rupees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(y)</td>
</tr>
<tr>
<td>5</td>
<td>0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.03</td>
</tr>
<tr>
<td>6</td>
<td>0.70</td>
</tr>
<tr>
<td>5</td>
<td>0.82</td>
</tr>
<tr>
<td>5</td>
<td>0.89</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
</tr>
<tr>
<td>6</td>
<td>0.66</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>1.69</td>
</tr>
<tr>
<td>7</td>
<td>0.70</td>
</tr>
<tr>
<td>7</td>
<td>0.48</td>
</tr>
</tbody>
</table>

(a) Determine the least square regression equation of $y$ on $x$.
(b) Describe the apparent relationship between age and price.
(c) Use the above regression equation to predict the price of a 3-year old car. (09)

Q.9 A sample size of $n = 81$ is taken from a population having a mean of 121 and standard deviation of 20. What is the probability that sample mean will lie between 120 and 128? (05)
Q.10  (a) Among 18 members of a cricket club, there are 2 wicket keepers and 5 bowlers. In how many ways can a team of 11 members be chosen so as to include only one wicket keeper and at least three bowlers? (04)

(b) Seventy five percent of youths 12-15 years of age have a blood pressure less than 136. What is the probability that a sample of 12 youths of that age group will include:

(i) Exactly 4 who have a blood pressure greater than 136?
(ii) At least 4 who have a blood pressure greater than 136? (05)

(c) The probability that a car will have a flat tyre while driving through a certain tunnel is 0.00006. Use the Poisson approximation to the binomial distribution, to find the probability that at least 2 out of 10,000 cars passing through this tunnel will have flat tyre. (04)

Q.11  A random sample of 16 Ph.D economists had an average income of Rs. 42,300 per month with a standard deviation of Rs. 4,000.

(a) Obtain a 98% confidence interval for the mean income of new Ph.D economists and interpret your result.

(b) How can you increase the precision and still maintain the same level of confidence? (07)

(THE END)
THE INSTITUTE OF CHARTERED ACCOUNTANTS OF PAKISTAN

Foundation Examinations  Autumn 2006

September 06, 2006

QUANTITATIVE METHODS                           (MARKS 100)  
Module A                                                                                                          (3 hours)

Q.1 (a) The selling price of an item is Rs. 300 and the gross profit is 15% of cost.  
Compute the cost.                                                                                                   (02)

(b) Find the value of x where  $3^{2x-1} = 6^{2x}$                                                                 (03)

(c) Determine the value of x by using quadratic formula where $9x^2 - 81 = 0$                                        (04)

Q.2 The cost of producing x units of a product is given by the function: 

\[ C(x) = 3.5x + 12,000 \]

In addition to the above, each unit is subject to a special levy of 20 paisa for earthquake victims. You are required to determine:

(a) the minimum number of items that should be produced and sold to avoid any loss, if each unit is sold for Rs. 6. (04)

(b) the minimum price per unit that should be charged to avoid any loss, if the estimated sales is 6000 units. (04)

Q.3 (a) A person deposited Rs. 100,000 in a bank for three years. The bank paid interest at the rate of 8% per annum compounded half yearly during the first year and at the rate of 12% per annum compounded quarterly during the last two years. Find his balance after three years. (04)

(b) How long will it take for a sum of money to double itself at 10% simple interest? (01)

Q.4 A machine costs a company Rs. 5,200,000 and its effective life is estimated to be 20 years. A sinking fund is set up for its replacement by a new machine at the end of its life time, when its scrap value will be Rs. 250,000 only. The price of the new machine is estimated to be 50% higher than that of the present one. Determine what amount should be set aside at the end of each year, if it earns 8% interest per annum compounded annually. (06)

Q.5 (a) Find $\frac{dy}{dx}$ if $y = e^{ax^3}$                                                                 (03)
If \( y = \left[ x + \sqrt{1 + x^2} \right]^m \) prove that \( \frac{dy}{dx} = \frac{my}{\sqrt{1 + x^2}} \).

Q.6 A company produces \( x \) units of output at a total cost of \( \frac{1}{3} x^3 - 18x^2 + 160x \), find:

(a) The output at which marginal cost is minimum.
(b) The output at which average cost is minimum.
(c) The output at which average cost is equal to marginal cost.

Q.7 (a) Simplify the following:
\[
\begin{bmatrix}
1 & 0 \\
-1 & 4
\end{bmatrix}
\begin{bmatrix}
1 & 4 \\
6 & 5
\end{bmatrix}
- \begin{bmatrix}
2 & 6 \\
5 & 0
\end{bmatrix}
\]

(b) Find \( x \) and \( y \), if matrix \( A \) has \( x \) rows and \( x+5 \) columns, matrix \( B \) has \( y \) rows and \( 11 - y \) columns and both \( AB \) and \( BA \) are defined for product.

(c) Solve for \( x \) and \( y \) if:
\[
\begin{bmatrix}
5 \\
2
\end{bmatrix}
= \begin{bmatrix}
15 \\
y
\end{bmatrix}
\]

Q.8 The duration of patients’ stay in a hospital were organized into a frequency distribution. The mean duration of stay was 28 days, the mode was 23 days and the median was 25 days. The standard deviation was 4.2 days.

Find the co-efficient of skewness. Is the distribution symmetrical, positively skewed or negatively skewed? Give reasons to support your answer.

Q.9 Calculate the cost of living index for the year 2005 taking year 2000 as the base, for the following data:

<table>
<thead>
<tr>
<th>Commodities</th>
<th>Expense in 2000 (Rupees)</th>
<th>Price (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000</td>
<td>2005</td>
</tr>
<tr>
<td>Food</td>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>Cloth</td>
<td>150</td>
<td>50</td>
</tr>
<tr>
<td>Education</td>
<td>450</td>
<td>60</td>
</tr>
<tr>
<td>Rent</td>
<td>750</td>
<td>40</td>
</tr>
<tr>
<td>Fuel</td>
<td>450</td>
<td>25</td>
</tr>
<tr>
<td>Others</td>
<td>300</td>
<td>35</td>
</tr>
</tbody>
</table>

Also calculate the purchasing power of money and real per capita income if current per capita income is estimated at Rs. 50,000 per year.

Q.10 (a) The heights and weights of six men are given below:

<table>
<thead>
<tr>
<th>Height (meters)</th>
<th>2.00</th>
<th>1.80</th>
<th>1.85</th>
<th>1.72</th>
<th>1.75</th>
<th>1.79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kgs)</td>
<td>85.0</td>
<td>78.0</td>
<td>80.0</td>
<td>74.0</td>
<td>75.0</td>
<td>76.0</td>
</tr>
</tbody>
</table>

(i) Determine both the lines of regression. Interpret the results as well.
(ii) Estimate weight when height is 1.70 meters.
(iii) Estimate height when weight is 70.0 kg.
(b) Different values of co-efficient of correlation ‘r’ are given below:

(i) \( r = 1 \)  
(ii) \( r = -1 \)  
(iii) \( r = 0 \)  
(iv) \( r = 0.90 \)  
(v) \( r = 0.10 \)  
(vi) \( r = -0.88 \)

Explain the type of relationship you would expect between ‘x’ and ‘y’ in each of the above cases.

Q. 11  

(a) A person is contesting for the directorship of two companies A & B. The chance of his success to become the director of company A is 70%, whereas the possibility of his winning the directorship of company B is 0.5. What is the probability that:

(i) He will become the director of both the companies.
(ii) He will become the director of at least one company.
(iii) He will not become the director in any of the two companies.

(b) A box contains six blue ink pens and four red ink pens. In how many ways can three pens be drawn in order to get at least one blue ink pen?

Q. 12  
The driving time of an executive while driving from his home to his office is normally distributed with a mean of 35 minutes and a standard deviation of 8 minutes. Assuming that total number of working days in a year are 300, you are required to calculate the number of days in which he is expected to drive to work:

(a) In less than or equal to 30 minutes?
(b) In more than or equal to 40 minutes?

Q. 13  

(a) The wholesaler of a toy has 1000 pieces in stock, out of which 200 pieces are slightly defective, mixed randomly with each other. A retailer buys one dozen toys. What is the probability that exactly 10 toys are defect free?

(b) The owner of a shopping mall wants to estimate the mean length of time shoppers spend in the mall. He wants the estimate to be within 5 minutes of the true mean time, with a 98% level of confidence. The estimate of the standard deviation of the length of time spent in the mall is 20 minutes. How large a sample is needed?

Q. 14  
Find a 95% confidence interval for the average life time of all the television sets produced by a company when a random sample of 25 television sets lasted an average of 10,000 hours with a standard deviation of 1,000 hours. Interpret your result.

Q. 15  
It is claimed that by using an automatic filling machine, the average amount of chips in each bag is 100 grams. A random sample of 36 bags of chips showed a mean weight of 101.25 grams with a standard deviation of 2.5 grams. Is the machine operating properly at 0.05 level of significance?

THE END
Q.1 (a) Find the value of x, using the laws of logarithms:
\[ 3^{2x-2} = \sqrt{2} \]  
(03)

(b) Factorize the given expression:
\[ 4x^4 + 81y^4 \]  
(03)

(c) Simplify the following to find the values of x:
\[ \frac{x+1}{x+2} + \frac{x+3}{x+4} = \frac{2x(x+1)}{x(x+2)(x+4)} \]  
(03)

(d) The sum of 12 terms of an arithmetic progression is 234 and the last term is 36. Find the common difference and the first term.  
(04)

Q.2 (a) Excel Corporation has issued bonds having face value of Rs. 500 million. They carry annual interest @10% compounded semi-annually. The bonds are redeemable in 2½ years. Interest as well as the face value will be paid on maturity. Find the amount that should be deposited each month into a sinking fund to be able to redeem all bonds on maturity, if the deposited amount earns interest @ 9% compounded monthly.  
(06)

(b) Compute effective rate of interest where nominal rate is 8% compounded quarterly.  
(02)

Q.3 (a) If \( y = 2x^2 - 6x + 9 \) and \( x = 3z + 5 \), find \( \frac{dy}{dz} \) by applying chain rule.  
(04)

(b) The cost of producing x units of calculators is denoted by:
\[ C(x) = 100 + 40x + \frac{1000}{x} \]
You are required to find:
(i) the marginal cost function;  
(ii) the total cost of 10 units; and  
(iii) the cost of producing the 100\(^{th}\) unit of calculator.  
(01)

(c) The demand of a product at a price of Rs. 5 per unit is 80 units. When the price is increased to Rs. 12 per unit the demand is reduced to 45 units. Determine the linear equation of the demand function in the form \( Q = f(p) \).  
(03)
Q.4 Solve the following system of equations by use of matrices:

\[
\begin{align*}
2x - y + 2z &= 6 \\
x - 2y + 3z &= 6 \\
3x - 3y - z &= -6
\end{align*}
\]

Q.5 A manufacturer produces two products P and Q which must pass through the same processes in the departments A and B having weekly production capacities of 240 hours and 100 hours respectively. Product P needs 4 hours in department A and 2 hours in department B. Product Q requires 3 hours and 1 hour respectively, in department A and B. Profit yields for product P is Rs. 700 and for product Q is Rs. 500. The manufacturer wants to maximize the profit within the given sets of limitations.

You are required to:

(a) Construct the objective function and all the constraints. 
(b) Plot the above constraints graphically and identify the feasible region. 
(c) Determine the combination of units which will give the maximum profit.

Q.6 (a) The data relating to the length of 30 leaves measured correctly to the nearest centimeter is shown below:

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>10 – 14</th>
<th>15 – 19</th>
<th>20 – 24</th>
<th>25 – 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3</td>
<td>8</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

Draw a histogram from the above data.

(b) The number of passengers carried by a boat during 25 trips is given below:

52 84 40 57 61 65 77 64 62 35 82 58 50 78 103 71 75 41 51 66 60 95 58 49 89

Construct a stem and leaf display with one digit leaves.

(c) Sohrab & Company increased the wages of its workers by 20% during the year 2006, whereas, the consumer price index changed from 240 to 270. Compute the increase / decrease in real wages.

Q.7 (a) Construct the scatter diagram for the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>9</th>
<th>2</th>
<th>12</th>
<th>7</th>
<th>16</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>18</td>
<td>11</td>
<td>16</td>
<td>9</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>

(b) Determine the coefficient of correlation between \( x \) and \( y \). 
(c) Calculate the coefficient of determination. 
(d) Interpret your results in (a), (b) and (c).

Q.8 (a) A firm installed two machines U and V, on January 1, 2007. The probability that the machines will break down during the first year of operations is 0.2 and 0.1 for machines U and V respectively. Compute the probability that one of the two machines will break down during the year.
(b) At a university, 40% of the students take Statistics without Mathematics and 25% take Mathematics without Statistics. 12% of the students take both. If a student is randomly selected from a Statistics class, what is the probability that he or she has also taken Mathematics?

Q.9 (a) From the past records it has been determined that 60% of a college’s students loan applications are approved. If ten applications are chosen at random, compute the following:
(i) probability that eight or more applications are approved;
(ii) number of applications that are expected to be approved; and
(iii) standard deviation of the number approved out of ten applications.

(b) The scores of an aptitude test are normally distributed with mean = 140 marks and \(\sigma = 25\) marks. Find the passing marks if the result is 67%.

Q.10 (a) A compressor manufacturer claims that average life of its compressors is 10,000 hours with a standard deviation of 100 hours and is normally distributed. Find the probability that a random sample of 25 compressors will have an average life between 9,980 hours and 10,050 hours.

(b) When properly adjusted, an automatic machine should produce parts that have a mean diameter of 25 millimeters (mm). Part diameters are normally distributed. The mean diameter of a sample of 10 parts is 25.02mm with sample standard deviation = 0.024. Perform a hypothesis test at 5% level of significance to evaluate whether the machine is working properly.

THE END
Q.1 (a) A briefcase selling for Rs. 1,500 was marked down by 20% for a special promotion. It was later marked down further by 10% of the promotional sales price. Compute the profit / loss in percentage if the cost is Rs. 1,000.

(b) Using quadratic formula, solve the following equation:

\[ x + 10 = 11x^2 - x + 1 \]

Q.2 A manufacturer produced 1200 units of a product in the first week. He wishes to plan his production and is considering the following options:

Option 1: Increase production by 80 units each week
Option 2: Increase production by 5% each week

Under each of the above options, compute the following:

(a) Output in the 20th week.
(b) Total output of the first 20 weeks.

Q.3 (a) The holder of a Promissory Note of Rs. 500,000 bearing interest at the rate of 9%, wishes to have it discounted, three months before the maturity date. The note was originally payable after one year of the date of issue. Compute the amount which the holder would receive if the discount rate is 10%.

(b) Hasan wants to save money over a period of ten years in order to meet the expenses to be incurred on higher education of his son. He has recently invested a sum of Rs. 100,000 and plans to save and invest Rs. 40,000 at the end of each year. Calculate the amount that will be available to him at the end of the 10th year, if he earns a profit of 8% each year.

Q.4 The total cost function of a product is defined by:

\[ TC = \frac{Q^3}{3} - 8Q^2 + 120Q + 10,000 \]

You are required to:

(a) Derive the marginal cost function.
(b) Determine the average cost of producing 1000 units.
(c) Determine the cost of producing the 6th batch, if the production is made in batches of 50 each.

Q.5 Differentiate the following with respect to x:

\[ y = \frac{\sqrt{2x}}{1 + x} \]
Q.6 (a) \[
A = \begin{bmatrix} 2 & 4 \\ 1 & 0 \\ 3 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 2 & 4 \\ 3 & 5 & -2 \\ -1 & 1 & 2 \end{bmatrix}
\]
Find the matrix C if \(A = 3BA - 2C\)  

(b) Cost of manufacturing a product consists of the cost of material, labour and overheads. Matrices \(X\) and \(Y\) represent the cost of material and labour respectively of three different models of the product produced in three different weeks. Overheads are 20% of the cost of labour.

\[
X = \begin{bmatrix} 20 & 10 & 40 \\ 30 & 20 & 50 \\ 40 & 20 & 30 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} 50 & 20 & 30 \\ 30 & 60 & 10 \\ 20 & 40 & 40 \end{bmatrix}
\]
Determine the sales price matrix if the producer earns a profit of 25% of total cost.  

Q.7 A research organization selected a sample of 30 visitors to a prestigious shopping mall. The data about the ages of the selected persons have been organized into the following table:

<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>Number of visitors</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 to 23</td>
<td>2</td>
</tr>
<tr>
<td>23 to 28</td>
<td>7</td>
</tr>
<tr>
<td>28 to 33</td>
<td>12</td>
</tr>
<tr>
<td>33 to 38</td>
<td>6</td>
</tr>
<tr>
<td>38 to 43</td>
<td>3</td>
</tr>
</tbody>
</table>

You are required to calculate the following:
(a) Range.
(b) Sample variance and sample standard deviation.
(c) Coefficient of variation.  

Q.8 For the following set of data:

<table>
<thead>
<tr>
<th>(x)</th>
<th>13</th>
<th>16</th>
<th>14</th>
<th>11</th>
<th>17</th>
<th>9</th>
<th>13</th>
<th>17</th>
<th>18</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>6.2</td>
<td>8.6</td>
<td>7.2</td>
<td>4.5</td>
<td>9.0</td>
<td>3.5</td>
<td>6.5</td>
<td>9.3</td>
<td>9.5</td>
<td>5.7</td>
</tr>
</tbody>
</table>

(a) Develop the estimation equation that best describes the data.
(b) Predict ‘\(y\)’ for \(x = 10\)  

Q.9 (a) A professor has decided to use weighted average for determining the final grades for his students. The home assignment will count for 20% of a student’s grade, quizzes 10%, the term paper 10%, the mid term 25% and the final 35%. From the following data, compute the final average for each student.

<table>
<thead>
<tr>
<th>Student</th>
<th>Home Work</th>
<th>Quizzes</th>
<th>Term Paper</th>
<th>Mid Term</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>85</td>
<td>89</td>
<td>94</td>
<td>87</td>
<td>90</td>
</tr>
<tr>
<td>B</td>
<td>78</td>
<td>84</td>
<td>88</td>
<td>91</td>
<td>92</td>
</tr>
<tr>
<td>C</td>
<td>94</td>
<td>88</td>
<td>93</td>
<td>86</td>
<td>89</td>
</tr>
</tbody>
</table>
(b) Consider the following box and whisker plot:

```
5  10  25  40  60  85  95
 5 15 25 35 45 55 65 75 85 95
```

From the above box and whisker plot, ascertain the following:
(i) Median.
(ii) Smallest and the largest values.
(iii) First and third quartiles.
(iv) Whether the distribution is symmetrical or skewed, along with justification.

Q.10 (a) If two dice are rolled, what is the probability that either the sum of the two will be seven or at least one of the dice will show the number 5.

(b) A loan officer in a bank estimates that the probability of default by an applicant is 0.025. Last month the bank approved 40 loan applications. Using Poisson distribution, find:
(i) The probability that 3 loans will be defaulted.
(ii) The probability that at least 3 loans will be defaulted.

(c) A manufacturer claims that average life of a certain type of product is 270 hours with a standard deviation of 25 hours. A sample of 12 units when tested showed a mean life of 255 hours. Using a significant level of 0.05, evaluate the manufacturer’s claim.

Q.11 (a) A training program has been designed to upgrade the supervisory skills. Supervisors take different number of hours to complete the program. A study of past participants indicates that the mean length of time spent on the program is 500 hours with a standard deviation of 100 hours. What is the probability that a candidate selected at random will require fewer than 580 hours to complete the program.

(b) A person tossed a coin 24,000 times and recorded 12,012 heads. You are required to calculate:
(i) Point estimate for proportion of heads in the population.
(ii) Standard error of the proportion.
(iii) 90% confidence interval for the population proportion and interpret the result.

(THE END)
Q.1 (a) Find the values of x and y in the given set of simultaneous equations:
\[
\frac{x+1}{y+1} = \frac{4}{5} \quad \text{and} \quad \frac{x-5}{y-5} = \frac{1}{2}
\]

(b) Find the equation of a straight line which intersects the y-axis at \( y = 4 \) and the x-axis at \( x = 8 \).

Q.2 (a) Using log 2 = 0.3010, log 3 = 0.4771 and log 5 = 0.6990, find the values of:
(i) \( \log 4.5 \)
(ii) \( \log 2\sqrt{5} \)

(b) If \( y = \sqrt{\ln 2x + e^{2x}} \) show that \( \frac{dy}{dx} = \frac{1 + 2xe^{2x}}{2xy} \)

Q.3 (a) How much should an individual deposit now to yield Rs. 600,000 at the end of five years in each of the following situations:
(i) At 10% simple interest
(ii) At 9% compounded half yearly

(b) Ashraf purchased a new car and made a down payment of Rs. 50,000. He is further required to pay Rs. 30,000 at the end of each quarter for five years. You are required to:
(i) Find the cash purchase price of the car, if the quarterly payments include 12% interest per annum compounded quarterly.
(ii) Find the total amount of interest Ashraf has to pay.

Q.4 (a) The revenue function for a product is:
\[ TR = 400q - 2q^2 \]
The average cost function is:
\[ AC = 0.2q + 4 + \frac{400}{q} \]

Determine the following:
(i) Level of output at which profit is maximized.
(ii) Price at which maximum profit occurs.
(iii) Amount of maximum profit.
(2)

(b) For the following set of inequalities:

\[ \begin{align*}
  x + 2y &\leq 8, \\
  3x + y &\leq 12 \\
  x + y &\leq 5, \\
  x &\geq 0, \\
  y &\geq 0
\end{align*} \]

Draw the graph and highlight the feasible region clearly indicating its boundaries. (07)

Q.5 Solve the following system of equations by Cramer’s Rule:

\[ \begin{align*}
  2x + 8y + 5z &= 5 \\
  x + y + z &= -2 \\
  x + 2y - z &= 2
\end{align*} \] (09)

Q.6 (a) Your Statistics teacher has given you an assignment to select ten students currently enrolled at your college and collect data for the following variables:

- \( X \) = Age of the students.
- \( Y \) = Number of family members.

You are required to answer the following:
(i) What is the population in the given scenario?
(ii) Is the population finite or infinite? Give brief reason to support your answer.
(iii) Classify the variables \( X \) and \( Y \) as discrete or continuous. (03)

(b) The following stem-and-leaf display shows the number of units produced per day:

```
3 | 8
4 | 
5 | 6
6 | 0 1 3 3 5 5 9
7 | 0 2 3 6 7 7 8
8 | 5 9
9 | 0 0 1 5 6
10| 3 6
```

Observe the above data and answer the following:
(i) For how many days, the data was collected?
(ii) What are the smallest and the largest values?
(iii) How many values are 80 or more?
(iv) List the actual values in second and fourth row.
(v) What is the middle value? (05)

(c) A binomial random variable has a mean of 200 and a standard deviation of 10. Find the values of \( n \) and \( p \). (04)

Q.7 (a) The two regression lines obtained in a correlation analysis are:

\[ 5x = 6y + 24 \quad \text{and} \quad 1000y = 768x - 3708 \]

Determine \( b_{xy} \); \( b_{yx} \) and the correlation coefficient ‘\( r \)’. (04)
(b) Students who finish the examinations more quickly than the rest are often thought to be smarter. The following set of data shows the score of 12 students and the order in which they finished their examination:

<table>
<thead>
<tr>
<th>Order of finish</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam score</td>
<td>90</td>
<td>78</td>
<td>76</td>
<td>60</td>
<td>92</td>
<td>86</td>
<td>74</td>
<td>87</td>
<td>86</td>
<td>60</td>
<td>78</td>
<td>68</td>
</tr>
</tbody>
</table>

Find the Spearman’s rank correlation co-efficient for the above data. (07)

Q.8 (a) Consider the following frequency distribution:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>10–19</th>
<th>20–29</th>
<th>30–39</th>
<th>40–49</th>
<th>50–59</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>14</td>
<td>24</td>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

Find the mean, variance and standard deviation. (05)

(b) The per capita income in a country has increased from $ 450 in the year 2000 to $ 960 in the year 2007. Taking base as 100 in the year 2000, the Consumer Price Index in 2007 stood at 160. Compute the real per capita income and the purchasing power of money, in the year 2007. (02)

Q.9 (a) A husband and wife were interviewed for two different posts in the same organization. The probability of husband’s selection is 1/7 and that of wife’s selection is 1/5. What is the probability that:

(i) Both of them will be selected.
(ii) None of them will be selected.
(iii) Only one of them will be selected. (04)

(b) A fair dice is rolled thrice. What is the probability that each time a six will appear. (02)

Q.10 (a) The following data were obtained from an experiment designed to estimate the reduction in blood pressure as a result of following a salt free diet for two weeks:

<table>
<thead>
<tr>
<th>Before</th>
<th>93</th>
<th>106</th>
<th>87</th>
<th>92</th>
<th>102</th>
<th>95</th>
<th>88</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>After</td>
<td>92</td>
<td>102</td>
<td>89</td>
<td>92</td>
<td>101</td>
<td>96</td>
<td>88</td>
<td>105</td>
</tr>
</tbody>
</table>

Assuming that the blood pressures of the population are normally distributed, find the 98% confidence interval for the mean reduction in blood pressure. (07)

(b) A random sample of size 16 has a mean of 53. The sum of squares of deviations of values from the mean is 150. Assuming the population values are approximately normally distributed, test the hypothesis that population mean is 56. (Take $\alpha = 0.05$) (07)

(THE END)
Q.1 (a) Find the value of \( x \) in the given equation:
\[ 8^{2x-4} = 16^x \]  
(03)

(b) A gardener has been given the task of digging an area of 800 square meters. It is expected that he will dig 10 square meters on the first day and on each successive day he will dig 1.2 times more than the area he dug on the previous day. Find the number of days the gardener will take to complete the task.  
(05)

Q.2 (a) The number of fishes in a lake is expected to increase at a rate of 8% per year. How many fishes will be in the lake in 5 years if 10,000 fishes are placed in the lake today?  
(03)

(b) Shahab has the opportunity to invest in a fund which earns 6% profit compounded annually. How much should he invest now if he wants to receive Rs. 6,000 (including principal) from the fund, at the end of each year for the next 10 years? How much interest he would earn over the period of 10 years?  
(06)

Q.3 (a) The average price of an item ‘R’ is directly related to the quantity ordered. The average price is Rs. 35 when 250 items are ordered and Rs. 55 if only 50 items are ordered. Identify the linear price function and calculate the price per item if a quantity of 115 is ordered.  
(04)

(b) A bank has provided two options to an investor:
- 11.1% profit compounded semi-annually.
- 11.0% profit compounded monthly.
Which option would you recommend?  
(03)

Q.4 (a) The demand function of an item is; \( P(x) = 100 - 0.01x \). Its cost function is; \( C(x) = 50x + 10,000 \). Determine the maximum profit that can be earned in the above situation and the price at which the profit will maximise.  
(07)

(b) If \( y = (x^2 - 1)^4 (x^2 + 1)^5 \); show that \( \frac{dy}{dx} = 2x(x^2 - 1)^3 (x^2 + 1)^4 (9x^2 - 1) \)  
(05)
Q.5  (a) Sketch the feasible region and identify the redundant constraint from the following set of inequalities:
\[ \begin{align*}
    x + y & \leq 6 \\
    5x + 3y & \leq 15 \\
    x & \leq 2 \\
    x, y & \geq 0
\end{align*} \]

(b) Solve the following system of equations by matrix inversion method:
\[ \begin{align*}
    2x - y + 3z &= 7 \\
    x + 3y - z &= 8 \\
    x + y - 4z &= 1
\end{align*} \]

Q.6  (a) Following are the ages of nine employees of an insurance company:
47, 28, 39, 51, 33, 37, 59, 24, 33
(i) Find the values of the first and the third quartile.
(ii) Calculate the quartile deviation.

(b) The mean annual salary of all employees in a company is Rs. 150,000. The mean annual salary of male and female employees is Rs. 162,000 and Rs. 102,000 respectively. Find the percentage of male and female employees in the company.

Q.7  (a) If two fair dice are rolled together, which of the following is more likely:
- Getting a total of 7 or more.
- Getting a total of 7 or less.

(b) In a group of 12 international referees, there are three from Africa, four from Asia and five from Europe. To officiate at a tournament, three referees are chosen at random from the group. Find the probability that:
(i) A referee is chosen from each Continent.
(ii) Two referees are chosen from Asia.
(iii) All the three referees are chosen from the same Continent.

Q.8  The data in the following table shows the monthly maintenance cost and the ages of nine similar machines operating in a factory:

<table>
<thead>
<tr>
<th>Machine</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (in months)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Cost (Rs in 000)</td>
<td>19</td>
<td>24</td>
<td>25</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>30</td>
<td>30</td>
<td>35</td>
</tr>
</tbody>
</table>

(a) Find the least square regression line of maintenance cost on age.
(b) Describe the apparent relationship between maintenance cost and age.
(c) Find the coefficient of correlation and interpret your result.
Q.9 (a) Sixteen cars of a specific model were selected at random. A test of fuel consumption gave a mean of 26.4 kilometer (km) per liter, with a standard deviation of 2.3 km per liter.

Assuming that the km per liter given by all cars of that model has a normal distribution, find a 99% confidence interval for population mean. Interpret your result.

(b) Compute Laspeyre’s Price Index for the following data using 2002 as base:

<table>
<thead>
<tr>
<th>Commodities</th>
<th>Price in 2002</th>
<th>Price in 2007</th>
<th>Quantity in 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>140</td>
<td>220</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>120</td>
<td>180</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>80</td>
<td>110</td>
<td>60</td>
</tr>
</tbody>
</table>

Q.10 (a) An advertising company wants to estimate with 97.5% confidence, the number of times a website is hit during an hour. It has determined that $\sigma = 26$.

How large a sample should the company take, if it wishes that the margin of error should not exceed 10?

(b) The life (in months) of a hair dryer is approximately normally distributed with mean 96 and a standard deviation of 18. You are required to determine the following:

(i) What percentage of the units sold will have to be replaced if the warranty period is five years?
(ii) The maximum warranty period if the manufacturer wants to limit the replacement to 1% of the units sold.

(THE END)
Q.1 (a) If \( \log \frac{xy}{x} = 0.9 \) and \( \log \frac{x}{y} = 0.5 \), find the following, without using a calculator/table:

(i) \( \log x \)

(ii) \( \log y \)

(b) Factorize the following expression to its simplest form:

\( ax^2 - ay^2 + bx^2 - by^2 \)

Q.2 (a) Ali walked a certain distance on the first of February 2009. On each successive day he walked 100 meters more than the previous day. If the total distance covered by him in 28 days is 51.8 km, find the distance covered by him on:

(i) the first day

(ii) the last day

(b) Asif works in a factory where wages are paid on weekly basis. The overtime rate is higher than the normal rate. In addition, the overtime in excess of eight hours per week is paid at a rate which is double the rate applicable to the first eight hours of overtime. The related data is as follows:

<table>
<thead>
<tr>
<th>Normal Hours</th>
<th>Overtime Hours</th>
<th>Pay Received (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st week of Feb’ 09</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>2nd week of Feb’ 09</td>
<td>40</td>
<td>15</td>
</tr>
</tbody>
</table>

Find the normal and overtime rates per hour.

Q.3 (a) A company is considering a project which requires an investment of Rs. 1,200,000 now and Rs. 300,000 at the end of the 1st year. It will earn Rs. 200,000 at the end of 2nd year and thereafter it will earn a fixed annual amount up to the 7th year. If interest rate is 11%, find the amount that the project should earn annually i.e. from year 3 to year 7 if the company desires to earn a net present value of Rs. 100,000.

(b) Younus Limited (YL) has borrowed an amount of Rs. 100,000 at an interest of 10% per annum compounded semi-annually. To pay off the loan at the end of four years, YL has created a sinking fund, which yields a return of 8% per annum compounded quarterly. Determine the amount which YL must deposit at the end of each quarter, in the sinking fund, to settle the loan at the end of four years?

Q.4 (a) If \( y = xe^{\log x} \), show that:

\[ \frac{dy}{dx} = y + e^{\log x}(1 + \log x) \]
(2)

(b) The cost of manufacturing x units of an item consists of the following:
   - Material: Rs. 2 per unit
   - Labour: Rs. \(x^2/90\) per unit
   - Overheads: Rs. 1000

Determine the number of units that should be produced in order to minimize the average cost. \(\text{(06)}\)

Q.5 (a) Find the value of \(K\), if:
\[
A = \begin{bmatrix} 0 & 3 \\ -2 & 5 \end{bmatrix}
\]
and
\[
KA^2 = 5A - 6I, \text{ where } I \text{ is an identity matrix of order 2.}
\]
\(\text{(05)}\)

(b) A furniture firm makes chairs and tables on three machines M1, M2 and M3. The detail of number of hours required per unit and total hours available on each machine is as follows:

<table>
<thead>
<tr>
<th>Machine</th>
<th>Chair</th>
<th>Table</th>
<th>Available time</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>3</td>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>M2</td>
<td>5</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>M3</td>
<td>2</td>
<td>6</td>
<td>60</td>
</tr>
</tbody>
</table>

Draw the graph of linear inequalities and indicate the feasible region by proper shading. \(\text{(08)}\)

Q.6 (a) The mean and standard deviation of a sample of 100 observations were found to be 104 and 4.7 respectively. Later, error was detected in three records as enumerated below:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Correct Figure (as per original record)</th>
<th>Amount Taken (for computation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>151</td>
<td>115</td>
</tr>
<tr>
<td>72</td>
<td>78</td>
<td>87</td>
</tr>
<tr>
<td>89</td>
<td>98</td>
<td>89</td>
</tr>
</tbody>
</table>

Find the correct mean and standard deviation. \(\text{(05)}\)

(b) Consider the following Stem-and-leaf display:

(i) For the data given above, determine:
   - Minimum value
   - Maximum value
   - Median
   - First quartile
   - Third quartile \(\text{(04)}\)

(ii) Make a Box and Whisker plot for the given data. \(\text{(02)}\)
Q.7 A company wants to assess the impact of advertising expenditures on its annual profit. The following table presents the information for eight years:

<table>
<thead>
<tr>
<th>Year</th>
<th>Advertising Expenditure</th>
<th>Annual Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>90</td>
<td>45</td>
</tr>
<tr>
<td>2002</td>
<td>100</td>
<td>42</td>
</tr>
<tr>
<td>2003</td>
<td>95</td>
<td>44</td>
</tr>
<tr>
<td>2004</td>
<td>110</td>
<td>60</td>
</tr>
<tr>
<td>2005</td>
<td>130</td>
<td>30</td>
</tr>
<tr>
<td>2006</td>
<td>145</td>
<td>34</td>
</tr>
<tr>
<td>2007</td>
<td>150</td>
<td>35</td>
</tr>
<tr>
<td>2008</td>
<td>140</td>
<td>30</td>
</tr>
</tbody>
</table>

(a) Construct the least square regression equation and predict the annual profit for the year 2009 if the advertising expenditure is budgeted at Rs. 160 million. (07)  
(b) Determine the coefficient of correlation and interpret your result. (04)  

Q.8  
(a) A dice is rolled twice. What is the probability that the number appearing in the first throw is greater than the number appearing in the later attempt? (03)  
(b) An industrial product is shipped in lots of 20. A sample of five units is selected by the buyer for inspection. The buyer rejects the lot if more than one defective item is observed in the sample. If a lot contains four defective items what is the probability that it would be accepted? (05)  
(c) People of Greenland have a mean height of 160 cm with a standard deviation of 15 cm. If a random sample of size 40 is taken, what is the probability that the sample mean height shall lie between 157 cm and 165 cm? (06)  

Q.9  
(a) An auditor claims that 10% of the customers’ ledger accounts contain mistakes. A random sample of 600 accounts was taken to test the accuracy of ledger accounts and mistakes were detected in 45 accounts. Using 5% level of significance, explain whether the sample result is consistent with the auditor’s claim. (07)  
(b) The city government has conducted a survey for assessing the inclination of citizens towards installing backup power arrangements in their homes. It has collected the following data:

<table>
<thead>
<tr>
<th></th>
<th>Owners</th>
<th>Tenants</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator only</td>
<td>220</td>
<td>200</td>
<td>420</td>
</tr>
<tr>
<td>UPS only</td>
<td>160</td>
<td>170</td>
<td>330</td>
</tr>
<tr>
<td>Generator + UPS</td>
<td>140</td>
<td>110</td>
<td>250</td>
</tr>
<tr>
<td>No backup</td>
<td>180</td>
<td>220</td>
<td>400</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>700</td>
<td>700</td>
<td>1400</td>
</tr>
</tbody>
</table>

Using chi–square test at 5% level of significance, assess the hypothesis that installation of backup power arrangements is independent of the types of residents i.e. owners or tenants. (07)  

(THE END)
Q.1 (a) If \( x^{2y} = \left( \sqrt{x} \right)^{y^2 + 2} \), find the value of \( y \). (03)

(b) Factorize the expression \( 16x^4 + 4x^2y^2 + y^4 \). (03)

(c) On August 1, 2009 a supplier increased the price of a product by 25%. On August 15, 2009, he decreased the price by 16%. If the price of the product on August 15, 2009 is Rs. 63, what was the price on July 31, 2009. (02)

Q.2 (a) A line passes through the point (3, 5) and has same values of x-intercept and y-intercept. Find the equation of the line. (03)

(b) A shopkeeper sold goods worth Rs. 3.0 million during 2008. If he is able to increase his sale by 15% annually, determine the year in which he would achieve annual sale of Rs. 25 million. (05)

Q.3 (a) A promissory note of Rs. 200,000 carries simple interest of 8% per annum. The note is payable at the end of 2 years. The holder of the note got it discounted 6 months before the maturity date and received an amount of Rs. 221,797. Compute the discount rate. (05)

(b) Shiraz acquired a new car worth Rs. 850,000 through a leasing company. He made a down payment of Rs. 200,000 and has agreed to pay the remaining amount in 10 equal semi-annual installments. The leasing company will charge interest @ 19% per annum, over the lease term. You are required to find:

(i) Amount of semi-annual installment. (05)

(ii) Total amount of interest that Shiraz will pay, over the term of the lease. (05)

Q.4 (a) The average cost function of a product is as follows:

\[
A(x) = 0.01x^2 - 30\sqrt{x} + 300 + \frac{60}{x}
\]

You are required to find the number of units at which the marginal cost will be minimum. (06)

(b) If \( y = \frac{(x - 1)(x + 2)}{\sqrt{x}} \), show that:

\[
\frac{dy}{dx} = \left( \frac{3x^2 + x + 2}{2x^2} \right)^{\sqrt{x}}
\] (04)
Q.5 (a) Using matrix algebra, find the value of ‘a’, if $|A| = 0$ for the following set of equations:

$$4x + 3y - z = 5$$
$$x - 2y + z = 4$$
$$ax + 12y - 5z = 1$$

(b) A pharmaceutical company has developed a formula to prepare a herbal medicine. The medicine can be produced by using either product X or product Y or a combination of both. From each milligram (mg) of X it can extract one unit of iron and two units of calcium and from each mg of Y it can extract one unit of iron and one unit of calcium. Each tablet of the medicine is required to contain:

- 5 to 7 units of iron
- 8 to 10 units of calcium

The cost of X is Rs. 6 per mg whereas Y costs Rs. 4 per mg.

You are required to:
(i) Construct the set of constraints and the objective function for cost minimization.
(ii) Draw the graph and identify the feasible region, clearly indicating its boundaries.
(iii) How many mg of each product should be used to produce the tablets at the lowest cost?

Q.6 (a) The following data represents the average monthly take-home salary of the employees of an organization:

<table>
<thead>
<tr>
<th>Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay (Rs.)</td>
<td>12,350</td>
<td>13,500</td>
<td>14,800</td>
<td>16,500</td>
</tr>
<tr>
<td>Price Index</td>
<td>110.1</td>
<td>122.3</td>
<td>137.6</td>
<td>160.2</td>
</tr>
</tbody>
</table>

(i) Compute the real wages for each of the above years.
(ii) Compute the amount of pay needed in 2008 to provide buying power equal to that enjoyed in 2006.

(b) The following data relates to salaries of the employees of a reputed cement manufacturing company:

<table>
<thead>
<tr>
<th>Salary (Rs in thousands)</th>
<th>Number of Employees</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 up to 20</td>
<td>120</td>
</tr>
<tr>
<td>20 up to 30</td>
<td>175</td>
</tr>
<tr>
<td>30 up to 40</td>
<td>100</td>
</tr>
<tr>
<td>40 up to 50</td>
<td>80</td>
</tr>
<tr>
<td>50 up to 60</td>
<td>44</td>
</tr>
<tr>
<td>60 up to 70</td>
<td>25</td>
</tr>
<tr>
<td>70 up to 80</td>
<td>17</td>
</tr>
</tbody>
</table>

(i) Draw a frequency polygon representing the above data.
(ii) Assuming that the mean and standard deviation of salaries of another company is Rs. 36,544 and Rs. 8,982 respectively, determine which company’s salaries are more evenly distributed.
Q.7  (a) Following data shows the marks obtained by 11 students in Mathematics and Physics:

<table>
<thead>
<tr>
<th>Mathematics</th>
<th>27</th>
<th>73</th>
<th>34</th>
<th>25</th>
<th>64</th>
<th>91</th>
<th>70</th>
<th>62</th>
<th>55</th>
<th>48</th>
<th>59</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics</td>
<td>67</td>
<td>62</td>
<td>41</td>
<td>21</td>
<td>74</td>
<td>85</td>
<td>66</td>
<td>49</td>
<td>55</td>
<td>44</td>
<td>68</td>
</tr>
</tbody>
</table>

Find the Spearman’s rank correlation coefficient for the above data and interpret your result.

(b) In order to determine the relationship between experience of its employees and their respective output, a company has gathered the following data:

<table>
<thead>
<tr>
<th>Experience in years</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output in %</td>
<td>30</td>
<td>35</td>
<td>44</td>
<td>43</td>
<td>46</td>
<td>50</td>
<td>45</td>
<td>48</td>
<td>39</td>
<td>34</td>
</tr>
</tbody>
</table>

(i) Determine the regression equation for output on experience.
(ii) Describe the apparent relationship between experience and output.
(iii) Predict the output of a 13 year experienced employee, using the regression equation.

Q.8  (a) During a T20 cricket match Naeem scored 47 runs in six overs with the help of five fours, four sixes and three singles. If a TV Channel were to show two of his scoring shots during its News Update and the shots were to be selected on random basis, find the probability of either of the following:

(i) On both the shots Naeem had scored different runs.
(ii) On one of the shots he had hit a six.

(b) The PABX system of Saleem Industries receives an average of two calls in every three minutes. Assuming an approximate Poisson distribution, what is the probability that five or more calls will be received during a period of nine minutes?

Q.9  (a) A cigarette manufacturer claims that the amount of nicotine in each cigarette is 7.5 mg. A random sample of 40 cigarettes was tested and found to have a mean nicotine content of 7.67 mg with a standard deviation of 0.6 mg. Test the manufacturer’s claim at 5% level of significance.

(b) A random sample of 20 boys was taken to estimate the 98% confidence interval for the mean weight of boys aged between 14 and 15 years. The 98% confidence interval was found to be 45.5 kg < µ < 51.3 kg. Find the sample mean and sample variance.

(THE END)
THE INSTITUTE OF CHARTERED ACCOUNTANTS OF PAKISTAN

Foundation Examinations    Spring 2010

March 2, 2010

QUANTITATIVE METHODS
Module A

(QRKS 100)   (3 hours)

Q.1 (a) The sum of a geometric series is 364 times its first term. If the common ratio is 3, find the number of terms in the series. (04)

(b) For positive integers x and y, find all the possible set of values satisfying the following equation:
\[2xy(x - 1) = 420\] (05)

Q.2 (a) Imran deposited Rs. 3,000 per month into a saving account for a year. He would deposit Rs. 5,000, Rs. 8,000 and Rs. 10,000 per month during second, third and fourth year respectively. If the bank offers 6% interest compounded monthly, find the total amount Imran would have saved at the end of four years. (10)

(b) Nosherwan Builders have launched an apartment project. Price of each apartment is Rs. 1,628,000. The buyer has to pay Rs. 200,000 at the time of booking and Rs. 34,000 per month for 3½ years to the builder. Possession will be given to buyers six months after the payment of last installment. If the amount of interest included in the price payable to the builder is Rs. 628,000, using logarithm, determine the annual rate of interest compounded monthly. (06)

Q.3 (a) If \(y = \frac{e^{2x}}{(x^2 + 1)^2}\), show that: \(\frac{dy}{dx} = \frac{2y(x - 1)^2}{x^2 + 1}\) (06)

(b) The cost and demand functions for manufacturing x units of a product per month are:
\[C(x) = 6x + 95,000\]
\[D(x) = 1680 - 3x\]

You are required to calculate:
(i) the number of units that should be produced per month to maximize the profit;
(ii) price per unit; and
(iii) maximum profit per month. (07)

Q.4 (a) Sketch the following inequalities and analyze the feasible region:
\[2x + 3y \geq 12\]
\[3x + 2y \leq 6\]
\[x, y \geq 0\] (04)
(b) Solve the following system of equations using Cramer’s rule:

\[
\begin{align*}
4x + 2y + z &= 6 \\
3x + 5y - 2z &= 4 \\
-x - 2y + z &= -1
\end{align*}
\]

Q.5 (a) A missile radar detection system consists of two radar screens. The probability that any of the radar screens will detect an incoming missile is 0.95. If a missile enters the detection space of this radar, what is the probability that at least one of the radar screens will detect it? (Assume that the radar detections are independent events.)

(b) Cool Tel is a large mobile service provider. It has conducted a study on 10,000 customers about the length of time they have to wait, at its customer care centres, before being facilitated by the Cool’s officer. The results of the study are as follows:

<table>
<thead>
<tr>
<th>Waiting Time (min)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of customers</td>
<td>380</td>
<td>1120</td>
<td>1680</td>
<td>1780</td>
<td>1960</td>
<td>1550</td>
<td>1200</td>
<td>330</td>
</tr>
</tbody>
</table>

You are required to:
(i) Construct the probability distribution for the above study.
(ii) Draw a histogram for the constructed probability distribution.
(iii) Determine the mean of the constructed probability distribution.
(iv) Calculate the probability that a customer must wait before being facilitated.
(v) What is the probability that a customer has to wait for less than two minutes before being facilitated?

Q.6 The following data shows the price and demand of a product at different points in time:

<table>
<thead>
<tr>
<th>Price (Rs.)</th>
<th>33</th>
<th>55</th>
<th>50</th>
<th>42</th>
<th>48</th>
<th>61</th>
<th>53</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (1000 kg)</td>
<td>91</td>
<td>60</td>
<td>59</td>
<td>65</td>
<td>61</td>
<td>49</td>
<td>42</td>
<td>91</td>
</tr>
</tbody>
</table>

(a) Determine the regression equation for the demand on price.
(b) Find the coefficient of correlation and coefficient of determination.
(c) Interpret the results obtained in (b) above.

Q.7 (a) Tara Electronics claims that its energy saver bulbs have an average life of 6500 hours. A consumer rights protection agency tested 15 such bulbs to check this claim. It found that the mean life of 15 bulbs was 6300 hours with a standard deviation of 200 hours. At the 5% significance level, assess the claim of Tara Electronics. Assume that life of such bulbs has an approximately normal distribution.

(b) In measuring reaction time, a psychologist estimated that the standard deviation is 0.05 second. How large a sample of measurements must he take in order to be 95% confident that the error in his estimate of mean reaction time will not exceed 0.01 second?
Q.8  (a) A local news channel has conducted an opinion poll for constructing more dams in the country. The poll result indicates that 70% of the participating viewers support the idea, 15% are against the idea and 15% are undecided. If a sample of six participating viewers is selected at random, determine the probability that:

(i) at least five viewers will support the idea.
(ii) less than two viewers will not support the idea.  

(b) A group of people was surveyed about their favourite car. The following results were obtained:

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Civic</td>
</tr>
<tr>
<td>Male</td>
<td>27</td>
</tr>
<tr>
<td>Female</td>
<td>26</td>
</tr>
</tbody>
</table>

At 5% level of significance, test the hypothesis that the choice of favourite car is independent of one’s gender.  

(THE END)
Q.1 (a) Companies A and B earned profits of Rs. 600,000 and Rs. 1,320,000 respectively during 2009. It is estimated that future annual profit would increase @ 8% and 5% respectively. In which year, the profit of both companies would be equal? (05 marks)

(b) Given that \( 8x^2 - 2xy - 3y^2 = 0 \); express \( 'y' \) in terms of \( 'x' \). (03 marks)

Q.2 Mr. Hamid plans to invest equal annual amounts in a bank for five years starting from January 1, 2011 in order to have the following amounts available with him:

- Rs. 1.0 million for the marriage of his daughter on January 01, 2018.
- Education expenses of his son consisting of four annual payments of Rs. 240,000 commencing from January 1, 2019.

If the bank agrees to pay interest @ 10% per annum compounded annually, calculate the amount of annual deposits which he would be required to invest. (08 marks)

Q.3 (a) If \( y = \frac{e^{2x}}{\sqrt{2x + 1}} \), show that \( \frac{dy}{dx} = \frac{y^3 (4x + 1)}{e^{4x}} \) (07 marks)

(b) Find out the coordinates of the relative maxima, minima and point of inflexion of the following function:

\[ y = \frac{1}{3} x^3 - 2x^2 + 3x - 9 \] (10 marks)

Q.4 Stable Limited manufactures two models of refrigerators. Deluxe Model requires 14 hours of Department A and 40 hours of Department B. Standard Model requires 7 hours of Department A and 30 hours of Department B. Each month, a maximum of 2100 hours are available in Department A and 8400 hours in Department B. The company makes a profit of Rs 5,000 on each Deluxe Model and Rs 4,000 on each Standard Model.

(a) Construct the set of constraints and the objective function for profit maximization.
(b) Draw the graph and identify the feasible region, clearly indicating its boundaries.
(c) How many refrigerators of each model should the company manufacture per day to maximize its profit? (09 marks)

Q.5 (a) Express the following system of equations in matrix form and determine their solution, if any, using matrix algebra.

\[
\begin{align*}
2x + 5y - z &= 20 \\
-3x - 2y + 7z &= 40 \\
-x + 3y + 6z &= 30
\end{align*}
\] (04 marks)

(P.T.O.)
(b) If \( M = \begin{bmatrix} 2 & -3 \\ 6 & 5 \end{bmatrix} \), \( N = \begin{bmatrix} -9 & 4 \\ -3 & 5 \end{bmatrix} \) and \( aM + bN = \begin{bmatrix} 61 & -44 \\ 63 & 15 \end{bmatrix} \)

where ‘a’ and ‘b’ are constants, find the values of ‘a’ and ‘b’. \( (04 \text{ marks}) \)

Q.6 (a) The following data represents the number of cancer patients admitted in a hospital over the last 14 years:

105, 60, 90, 110, 95, 140, 80, 70, 130, 90, 120, 75, 115, 85

(i) Find out the five numbers summary.
(ii) Draw a labelled box and whisker plot and define its skewness. \( (07 \text{ marks}) \)

(b) The following results were obtained in an IQ test taken by 50 boys and 60 girls:

<table>
<thead>
<tr>
<th></th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum x_1 )</td>
<td>4,910</td>
<td>( \sum x_2 )</td>
</tr>
<tr>
<td>( \sum x_1^2 )</td>
<td>483,515</td>
<td>( \sum x_2^2 )</td>
</tr>
</tbody>
</table>

Find the 90% confidence interval for the difference between the IQ of boys and girls. \( (07 \text{ marks}) \)

Q.7 (a) If the current year’s weighted index is 5% higher than the base year and Fisher’s Ideal Index Number is 250, find out the Laspeyre’s Price Index Number and Paasche’s Price Index Number. \( (04 \text{ marks}) \)

(b) The quantities sold by T&P Limited during the past seven months are as follows:

<table>
<thead>
<tr>
<th>Product x</th>
<th>11</th>
<th>20</th>
<th>04</th>
<th>zero</th>
<th>18</th>
<th>07</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product y</td>
<td>15</td>
<td>02</td>
<td>32</td>
<td>35</td>
<td>05</td>
<td>28</td>
<td>10</td>
</tr>
</tbody>
</table>

(i) Determine the regression equation for product y on x.
(ii) Calculate the coefficient of correlation and determination and interpret the results. \( (12 \text{ marks}) \)

Q.8 (a) Arshad’s journey from home to office takes a mean time of 21 minutes, with a standard deviation of 2.5 minutes. His journey back home takes a mean time of 28 minutes with a standard deviation of 5.5 minutes. Find the mean and standard deviation of the time taken by him on his two-way journey. \( (04 \text{ marks}) \)

(b) Out of every 4,000 shirts made at a garment factory, 22 are defective. Using Poisson distribution, find the probability that a lot of 300 shirts contain:

(i) more than 3 defective shirts
(ii) less than 2 defective shirts \( (07 \text{ marks}) \)

Q.9 Find out the probability that a random sample of size 36, selected with replacement, from the population 3, 5, 1, 7, 3, 6, 2, 4, 5, 2 will yield a sample mean greater than 3.2 but less than 4.5. \( (09 \text{ marks}) \)

(THE END)
Q.1 (a) Find the value of x such that:
\[ 4^{x+1} = 2 - 7(2^x) \]  
(05 marks)

(b) The y-intercept of a line has the same value as its gradient. If this line cuts the curve \( y = x^3 - 2x^2 - 3x - 8 \) at \( x = 3 \), find the equation of the line.  
(04 marks)

(c) Mr. Khan deposited an amount into a bank which will be doubled in eight years.
(i) Find the rate of interest considering that the amount is compounded annually.
(ii) How many years will it take for an amount to triple at the above calculated rate of interest?  
(05 marks)

Q.2 (a) Asif purchased a car for Rs. 360,000. The amount is payable in forty monthly installments which are in arithmetic progression. After paying thirty installments one third of the amount would remain unpaid. Calculate the amount which Asif would be required to pay as the 35th installment.  
(05 marks)

(b) Meena has invested Rs. 700,000 in an investment scheme. In return, she would receive Rs. 74,587 semi-annually in arrears, for six years. She would not receive any amount afterwards. Find the nominal and effective rate of return of the scheme.  
(05 marks)

Q.3 (a) If \( y = (1 + e^{2x})^2 \), show that:
\[ \frac{d^2y}{dx^2} = 6 \frac{dy}{dx} - 8y + 8 \]  
(04 marks)

(b) Sketch the feasible region and identify the point of optimal solution for the function \( z = 2x^2 + 5y \) subject to the following constraints.
\[ x - 3y \leq 0 \]
\[ x + 3y \geq 150 \]
\[ y \leq 50 \]
\[ x, y \geq 0 \]  
(07 marks)

Q.4 (a) Solve the following set of equations using Crammer’s rule or matrix inversion method.
\[ 2x - y + z + 1 = 0 \]
\[ 3x + 2y + 2z - 8 = 0 \]
\[ -x + 2y - z - 1 = 0 \]  
(08 marks)

(b) Find the co-ordinates of the relative minima and/or maxima of the following function:
\[ y = e^{2x} + 2e^x - 4x \]  
(07 marks)
Q.5 The following data shows the weight (in grams, rounded to the nearest gram) of 35 randomly picked oranges from a farm.
155, 161, 164, 166, 168, 170, 172, 173, 175, 177, 178, 178, 179, 181, 182, 182, 184, 186, 188, 189, 192, 195, 196, 197, 198, 203, 206, 208, 209, 210, 214, 218, 221, 243

(a) Find the median and mean from the above data.
(b) Group the data in the form of a table with class intervals and identify the modal class.
(c) Draw a histogram from the above grouped data.
(d) Suggest whether the data is positively skewed, negatively skewed, or symmetrical.
(e) Construct a stem-and-leaf display for the given data. 

(12 marks)

Q.6 (a) Find the coefficient of correlation between x and y if:
Regression line of \( x \) on \( y \) is: \( 5x - 4y + 2 = 0 \)
Regression line of \( y \) on \( x \) is: \( x - 5y + 3 = 0 \)

(04 marks)

(b) The average runs scored by seven leading test cricketers during the year 2010 are given below:

| Average runs scored in 1st innings (x) | 46 | 73 | 68 | 79 | 49 | 43 | 81 |
| Average runs scored in 2nd innings (y) | 66 | 31 | 45 | 26 | 58 | 63 | 35 |

Find the Spearman’s rank correlation coefficient for the runs scored in first and second innings and interpret your result.

(06 marks)

Q.7 (a) A problem in mathematics is given to three students A, B and C. Their respective probability of solving the problem is 1/2, 1/3 and 1/4. Find the probability that at least two of them will solve the problem.

(05 marks)

(b) There are six positive and eight negative numbers. Four numbers are chosen at random and multiplied. What is the probability that the product is a positive number?

(04 marks)

Q.8 A college has 500 students. A survey was carried out on 22 students chosen at random, to find out the average number of siblings. The following data was obtained:
2, 1, 3, 2, 0, 1, 2, 4, 0, 1, 1, 0, 1, 3, 2, 0, 1, 2, 2, 1, 0, 1

(a) Find the 90% confidence interval for the mean number of siblings of the college students.

(07 marks)

(b) Using the point estimates of the mean and standard deviation, calculate the probability that if a sample of 50 students is chosen at random, the average number of siblings of the sample would be more than 1.5.

(05 marks)

Q.9 A dice was tossed 144 times and following outcomes were recorded:

<table>
<thead>
<tr>
<th>Faces</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Occurrence</td>
<td>22</td>
<td>23</td>
<td>27</td>
<td>25</td>
<td>26</td>
<td>21</td>
</tr>
</tbody>
</table>

Using chi-square test at 5% level of significance, assess the hypothesis that the dice is fair.

(07 marks)

(THE END)
(All questions are compulsory)

Q.1 (a) During annual clearance sale, Independent Departmental Store (IDS) reduced the prices of a product Z by 20%. 40% of the sale of Z was made during the clearance sale and the profit earned thereon was 25% of cost. Find the profit percentage (on cost) which IDS earned on product Z during the whole year.

(b) The cost of production of a product in rupees is: \( C = 15x + 9,750 \) where \( x \) is the number of items produced. If selling price of each item is Rs. 30, find the sales quantity at which there would be no profit or loss.

(c) Find the value of \( x \) if, \( \ln 3 + 2\ln x = \ln(x + 2) \)

Q.2 (a) Bashir has chosen to receive a pocket money of Rs. 1,200 per week. He was also offered an alternative by which he would have received Re. 1 on the first day of the week, Rs. 3 on the second day, Rs. 9 on the third day and so on. Has he taken a wise decision?

(b) Shiraz borrowed Rs. 120,000 for eight months at 15% simple interest. Compute the annual rate of interest, compounded monthly, which would result in the payment of the same amount of interest.

Q.3 (a) If \( y = \frac{u}{u + 1} \) and \( u = 5x^2 - 1 \), find \( \frac{dy}{dx} \) at \( x = 2 \)

(b) The cost of manufacturing \( x \) units of a product consists of the following:

- Labour Rs. \( 0.03x^2 \);
- Material Rs. 220x;
- Overheads Rs. 50,000

The demand function of the same product is \( D(x) = 500 - 0.04x \)

Required:
(i) Revenue function and the marginal revenue function.
(ii) Profit function and the quantity at which profit is maximised.
(iii) Unit price corresponding to maximum profit.

Q.4 A company has two plants, \( P_1 \) and \( P_2 \) in two different cities. Each plant manufactures two products \( x \) and \( y \). Daily production of \( P_1 \) is 50 units of \( x \) and 30 units of \( y \). \( P_2 \) produces 20 units of \( x \) and 15 units of \( y \) per day. Each unit of \( x \) requires 40 hours of operational, 10 hours of technical and 5 hours of managerial staff. While each unit of \( y \) requires 50 hours of operational, 12 hours of technical and 7 hours of managerial staff.

Required:
(a) Construct a matrix \( P \) depicting the relationship between plants and products and a matrix \( S \) depicting the relationship between products and various categories of staff as described above.

(b) (i) Using the matrices constructed in part (a), compute the matrix \( R = PS \).
(ii) Based on matrix \( R \), determine the number of total hours worked by each category of staff, at each plant.
Q.5 Solve the following set of inequalities by graphical method and find the feasible region:
\[ x + y \geq 8; \quad 2x + y \geq 12; \quad x + y \leq 10; \quad x \geq 0 \text{ and } y \geq 0 \]  
(06 marks)

Q.6 (a) A survey of 316 randomly selected patients in a hospital, produced the following data:

<table>
<thead>
<tr>
<th>Hospital stays (in days)</th>
<th>Frequency (patients)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–3</td>
<td>06</td>
</tr>
<tr>
<td>4–6</td>
<td>12</td>
</tr>
<tr>
<td>7–9</td>
<td>110</td>
</tr>
<tr>
<td>10–12</td>
<td>103</td>
</tr>
<tr>
<td>13–15</td>
<td>42</td>
</tr>
<tr>
<td>16–18</td>
<td>25</td>
</tr>
<tr>
<td>19–21</td>
<td>13</td>
</tr>
<tr>
<td>22–24</td>
<td>04</td>
</tr>
<tr>
<td>25–27</td>
<td>01</td>
</tr>
</tbody>
</table>

Required:
(i) Calculate the mean and standard deviation of the above data.  
(04 marks)
(ii) Based on Chebyshev’s theorem, determine the minimum number of patients who would stay between 3 to 19 days if 6,000 patients are admitted to the hospital in a year.  
(03 marks)

(b) The following data represents the prices and consumption of fuel related products:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Prices per litre (Rs.)</th>
<th>Quantity in millions of litres</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diesel</td>
<td>30 90</td>
<td>150 200</td>
</tr>
<tr>
<td>Petrol</td>
<td>55 85</td>
<td>200 240</td>
</tr>
<tr>
<td>Kerosine</td>
<td>15 55</td>
<td>40 30</td>
</tr>
<tr>
<td>CNG</td>
<td>25 50</td>
<td>100 250</td>
</tr>
</tbody>
</table>

Calculate the price indices of Laspeyre, Paasche and Fisher, for the year 2010.  
(06 marks)

Q.7 (a) A box contains 10 items, 3 of which are defective. If 4 are selected at random, without replacement, find the probability that at least 2 are defective.  
(04 marks)

(b) In a T20 cricket match between Falcon Club (FC) and Eagle Club (EC), the probability to win by (FC) is 0.4. In a series of five T20 matches, find the probability that FC would win:
(i) Exactly two matches.  
(ii) At least two matches.  
(iii) Less than four matches.  
(06 marks)

Q.8 The following data shows the height and weight of eight men working in an organisation who were selected at random:

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td>155</td>
<td>63</td>
</tr>
<tr>
<td>161</td>
<td>67</td>
</tr>
<tr>
<td>166</td>
<td>70</td>
</tr>
<tr>
<td>170</td>
<td>72</td>
</tr>
<tr>
<td>174</td>
<td>74</td>
</tr>
<tr>
<td>182</td>
<td>76</td>
</tr>
<tr>
<td>190</td>
<td>78</td>
</tr>
</tbody>
</table>

Required:
(a) Determine the regression equation of weight over height and interpret your result.  
(07 marks)
(b) Determine the coefficient of correlation and coefficient of determination and interpret your result.  
(05 marks)

Q.9 (a) The following data pertains to a sample taken from a normal population:
5, 10, 8, 11, 12, 6, 15, 13, 10

Required:
(i) Find the point estimate for the population mean.  
(03 marks)
(ii) Construct 95% confidence interval for population mean and interpret your result.  
(06 marks)

(b) A manufacturer claims that after using an automatic bottling plant, the average quantity in each bottle produced on its plant is 250 ml. A random sample of 25 bottles showed a mean quantity of 242 ml with a standard deviation of 18 ml. Test the manufacturer’s claim using a significance level of 0.05.  
(06 marks)

(THE END)
Q.1  (a) Factorize $f(x)$ by completing the square where:
$$f(x) = x^4 - 2x^2y^2 - 8y^4$$
Find the possible value of $x$, where $f(x) = 0$ and $y = 5$  \((05\text{ marks})\)

(b) Find the sum of all integers between 170 and 1000 which are exactly divisible by 8.  \((05\text{ marks})\)

(c) Solve the following equation:
$$(2e^{5x} + 5e^x)(2e^{2x} - 11) = -55e^x$$
\((06\text{ marks})\)

Q.2  (a) Ali would require a sum of Rs. 300,000 after three years from now and a sum of Rs. 500,000 after five years from now, for the purpose of education of his son. He is planning to deposit quarterly amounts in an investment scheme to get the desired amounts at the required time. If the rate of interest is 12% compounded quarterly, what amount should Ali deposit at the start of each quarter?  \((05\text{ marks})\)

(b) Hayyan invested an amount of Rs. 400,000 in an investment scheme and got Rs. 545,881 at the end of three years. Find:

(i) The effective annual rate of interest if interest was compounded monthly.
(ii) The nominal rate if interest was compounded quarterly.  \((06\text{ marks})\)

Q.3  (a) Find $dy/dx$ where
$$y = \frac{(x + 3)(x - 2)}{\sqrt{x}}$$
\((05\text{ marks})\)

(b) The demand function for a firm’s product is $q = 150,000 - 75p$ where $q$ equals the number of units demanded and $p$ equals the price in rupees.

(i) Determine the price which should be charged to maximize total revenue.
(ii) What is the maximum revenue that the firm can earn?
(iii) How many units are expected to be demanded when the firm is earning maximum revenue?  \((07\text{ marks})\)

Q.4  (a) Sketch the feasible region and identify the redundant constraints from the following set of inequalities:

(i) $x + y \leq 6$  \hspace{1cm} (ii) $5x + 3y \leq 15$  \hspace{1cm} (iii) $x \leq 2$  \hspace{1cm} (iv) $x, y \geq 0$  \hspace{1cm} \((07\text{ marks})\)

(b) If $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$
Prove that $B$ is an inverse of $A$.  \((04\text{ marks})\)

Q.5 Company A and B are cement manufactures. The quantity of cement in bags produced by company A has a mean of 50 kg and standard deviation of 0.24 kg, while quantity in bags of company B has a mean of 49.8 kg and standard deviation of 0.75 kg.

If random samples of 30 bags from each company are taken, find the probability that the difference between means of the two samples is more than 0.25 kg.  \((07\text{ marks})\)
Q.6  The following table shows the actual monthly wages and the real wages of a worker over the previous years:

<table>
<thead>
<tr>
<th>Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages (Rs.)</td>
<td>11,000</td>
<td>12,000</td>
<td>13,500</td>
<td>14,800</td>
<td>16,500</td>
<td>19,000</td>
</tr>
<tr>
<td>Real Wages (Rs.)</td>
<td>11,000</td>
<td>10,800</td>
<td>11,300</td>
<td>11,100</td>
<td>10,550</td>
<td>10,900</td>
</tr>
</tbody>
</table>

(a) Compute the price index for the years 2006 to 2010, rounded to one decimal place, taking 2005 as the base year. 

(b) If the price index for the year 2011 is 191.2, calculate the amount of wages whose buying power would be the same as that of the year 2007?

Q.7  Following table shows the runs scored by a batsman in limited over matches during a calendar year:

<table>
<thead>
<tr>
<th>Range of Runs (x)</th>
<th>0 – 09</th>
<th>10 – 19</th>
<th>20 – 29</th>
<th>30 – 39</th>
<th>40 – 49</th>
<th>50 – 59</th>
<th>60 – 69</th>
<th>70 – 79</th>
<th>80 – 89</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Matches (f)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Draw a histogram based on the above frequency distribution. 

(b) Compute the inter-quartile range for the above distribution.

Q.8  Given below is a set of data for two variables:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>10</td>
</tr>
<tr>
<td>45</td>
<td>16</td>
</tr>
<tr>
<td>63</td>
<td>18</td>
</tr>
<tr>
<td>53</td>
<td>20</td>
</tr>
<tr>
<td>56</td>
<td>19</td>
</tr>
<tr>
<td>43</td>
<td>16</td>
</tr>
</tbody>
</table>

(a) Determine the regression line for x on y.

(b) Find the coefficient of correlation and interpret the result.

(c) Identify the point where the regression line for x on y will intersect the regression line for y on x.

Q.9  The following table shows the number of courses being taken by 100 students in a college:

<table>
<thead>
<tr>
<th>Number of Courses</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>

(a) Construct a probability distribution table for the number of courses taken by a student picked up at random from among the above 100 students.

(b) If forty students are selected at random, with replacement, find the probability that the mean number of courses being taken by them is more than 5.3.

(c) If two students are selected at random, with replacement, find the probability that the mean number of courses being taken by them is less than 4.7.

Q.10 (a) During safety and security drive of the Health Department, 190 private hospitals of a large city were surveyed. It was found that 79 hospitals were in violation of sanitary standards, 86 were in violation of security standards and 58 were in violation of both. If a private hospital is chosen at random, find the probability that it is in compliance with:

(i) sanitary standards but not with security standards.
(ii) both security and sanitary standards.

(b) A bag contains four red balls, three green balls, one blue ball and one yellow ball. Two balls are drawn out from the bag at random, without replacement. Find the probability that:

(i) both the balls are of the same colour.
(ii) both are of different colour.
(iii) at least one ball is green.

(THE END)
Q.1 (a) Sadiq has recently been appointed as an accountant in XYZ Ltd. He intends to set aside an amount of Rs. 10,000 from his salary received in the first month which would be increased by Rs. 500 in each of the subsequent months.

You are required to determine the number of months it would take for his aggregate savings to exceed Rs. 500,000.

(b) ABC Model School earned a net income of Rs. 2 million per annum during 2011 whereas the cost incurred on each student was 60% of the fee charged by the school. Compute the net profit for the year 2015 if it is projected that the number of students would increase by 5% per annum whereas the fee charged and the cost incurred per student would increase each year by 10% and 6% respectively.

Q.2 (a) An amount of Rs. 20,000 is due in three months. What is the present value if it includes simple interest @ 8%?

(b) Amin deposited Rs. 25,000 every six months in a fund earning interest at 6% compounded semi annually. The first deposit was made when he was 48 years of age and the last deposit was made when he was 55 and at that time the entire amount was invested in another fund which yielded 8% per annum. How much amount will he have in the fund, when he retires at the age of 60 years?

(c) The population of a country increases at the rate of 3% per annum. How many years will it take to double itself?

Q.3 (a) The cost and price functions for $x$ units of a product per month are:

\[ C(x) = \frac{1}{9}x^2 + 6x + 200 \quad \text{and} \quad \]

\[ P(x) = \frac{1}{2}(850 - x) \]

You are required to calculate:

(i) The number of units that should be produced per month to maximize the profit.
(ii) Price per unit.
(iii) Maximum profit per month.

(b) If $y = xe^y$, then show that:

\[ \frac{dy}{dx} = \frac{y}{x(1 - y)} \]

Q.4 (a) Solve the following set of equations using Cramer’s Rule:

\[ x + y - z = 4 \]

\[ 2x + z = 7 \]

\[ 3x - 2y = 5 \]
(b) Plot the following inequalities on graph and identify the feasible region:
\[ x, y \geq 1; \quad x, y \leq 4 \quad \text{and} \quad x \geq y \]  
(04 marks)

Q.5 (a) A sample survey conducted by an organization obtained the following data on the average number of times that persons in the various age groups visit a physician each year:

<table>
<thead>
<tr>
<th>Age group (years)</th>
<th>Number of persons in the sample</th>
<th>Mean number of visits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 5</td>
<td>50</td>
<td>2.1</td>
</tr>
<tr>
<td>5 — 20</td>
<td>115</td>
<td>1.6</td>
</tr>
<tr>
<td>21 — 60</td>
<td>155</td>
<td>2.6</td>
</tr>
<tr>
<td>61 and over</td>
<td>90</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Calculate the mean number of visits to the physician.  
(03 marks)

(b) Draw a box plot for the following data:
83, 63, 65, 70, 57, 66, 86, 75, 72, 64, 70, 60, 67, 78, 62

Explain whether the data is positively skewed or negatively skewed.  
(07 marks)

Q.6 The following table presents the mean height of children by their age:

<table>
<thead>
<tr>
<th>Age in months (x)</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height in inches (y)</td>
<td>30.0</td>
<td>30.3</td>
<td>30.7</td>
<td>30.8</td>
<td>31.0</td>
<td>31.4</td>
<td>31.5</td>
<td>31.9</td>
<td>32.0</td>
<td>32.2</td>
<td>32.6</td>
<td>32.9</td>
</tr>
</tbody>
</table>

(a) Establish the least square regression line of y on x.  
(06 marks)

(b) Compute the co-efficient of correlation and co-efficient of determination and interpret your result.  
(05 marks)

(c) Explain whether it would be appropriate to predict the mean height of a person of 20 years of age by using regression line obtained in (a) above.  
(02 marks)

Q.7 (a) A company owns two machines. Machine A produces red toys whereas machine B produces blue toys. Defective units produced by machine A and B average 25% and 30% respectively and their daily production is 20 and 30 units respectively.

If the daily production is mixed and three toys are selected at random, without replacement, find the probability that the first toy is a defective unit, the second toy is not defective whereas the third toy is defective and of blue colour.  
(05 marks)

(b) The average number of traffic accidents on a certain section of highway is two per week. Assuming that the number of accidents follows a Poisson Distribution, find the probability of at most 3 accidents on this section of highway during a two week period.  
(05 marks)

Q.8 (a) A television channel has claimed that on average a teenager spends 7 hours each month on viewing its entertainment programs.

An advertising agency wants to carry out a survey to assess the reasonableness of the above claim. Determine the size of the sample which would be needed in order to assert with 99% confidence that the error in the above claim does not exceed 30 minutes. Assume that \( \sigma = 3.2 \) hours.  
(03 marks)
(b) A researcher wants to test on the basis of a random sample of size \( n = 7 \), whether the fat contents of a certain kind of processed food does not exceed 30%. What would he conclude at the 0.01 level of significance, if the sample values (in percentages) are 31.5, 30.3, 31.1, 30.7, 29.9, 29.6 and 31.8.

Q.9 The maximum speed limit on a busy road is 60 km/h. Congestion results in much slower actual speeds. A random sample of 57 vehicles gave an average speed of 23.2 km/h with a standard deviation of 0.3 km/h.

(a) Determine the point estimate of the standard deviation of the population.
(b) Estimate the standard error of the mean.
(c) What are the upper and lower limits of the confidence interval for the mean speed, given a confidence level of 95%?
Q.1 (a) A car was moving at a speed of 135 km per hour. When brakes were applied, the speed of the car reduced to 43.2 km per hour in five seconds. Find the rate of decline in the speed of the car per second, if the percentage decrease after each second was the same. 

(b) Find the value of x:
   (i) \( \frac{(x + 3)}{(x - 2)} - \frac{8}{3} = \frac{(x + 2)}{(x - 1)} \)
   (ii) \( e^{2x} - 1 = 0 \)

Q.2 (a) The difference between simple and compound interest on a certain amount of money for 8 years at 14% per annum is Rs. 12,500. Find the amount.

(b) Ali paid Rs. 34,434 per month for three years to pay back a bank loan. Calculate the amount borrowed by Ali and interest paid thereon, if interest was charged at the rate of 14.5% per annum on the outstanding amount, on a monthly basis.

(c) The cost of wood used by a furniture manufacturer for making a table amounts to Rs. 7,000. Other costs incurred by him amount to 30% of the cost of wood. What price shall he charge, if he wishes to earn a profit of 12.5% of the selling price?

Q.3 (a) If \( y = (x^2 - 1)^4(x^2 + 1)^5 \), show that:
\[
\frac{dy}{dx} = 2x (x^2 - 1)^3(x^2 + 1)^4 (9x^2 - 1)
\]

(b) At selling price of Rs. 38 per unit, monthly sale of a product is estimated at 10,200 units. However, if selling price is increased by Rs. 9 per unit, it is expected that monthly sale would reduce to 8,400 units. The total cost function of the product is:
\[
C(x) = 15,000 + 18x, \text{ where } x \text{ is the number of units.}
\]

(i) Determine the price function, assuming it is linear.
(ii) Calculate the maximum monthly profit that can be earned.

Q.4 (a) Solve the following system of equations by using matrix inversion method:

(i) \( x + y - z = 2 \)  (ii) \( 2x - y + 3z = 17 \)  (iii) \( 3x + 2y - 4z = -3 \) 

(b) For the following set of inequalities, draw the graph and highlight the feasible region clearly indicating its boundaries:

(i) \( x + y \geq 7 \)  (ii) \( 2x + y \geq 10 \)  (iii) \( x + y \leq 8 \)  (iv) \( x, y \geq 0 \)

Q.5 (a) Age distribution of employees in Young Corporation is as follows:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>22-26</th>
<th>26-30</th>
<th>30-34</th>
<th>34-38</th>
<th>38-42</th>
<th>42-46</th>
<th>46-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of employees</td>
<td>6</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Find coefficient of variation of age of employees.
(b) Following data has been gathered from a survey:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>2010 Price (Rs.)</th>
<th>2011 Price (Rs.)</th>
<th>2012 Price (Rs.)</th>
<th>2010 Quantity (kg)</th>
<th>2011 Quantity (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>64</td>
<td>75</td>
<td>80</td>
<td>270</td>
<td>276</td>
</tr>
<tr>
<td>Beta</td>
<td>40</td>
<td>45</td>
<td>41</td>
<td>124</td>
<td>118</td>
</tr>
<tr>
<td>Gamma</td>
<td>18</td>
<td>21</td>
<td>20</td>
<td>130</td>
<td>121</td>
</tr>
<tr>
<td>Eta</td>
<td>58</td>
<td>68</td>
<td>56</td>
<td>185</td>
<td>267</td>
</tr>
</tbody>
</table>

(i) Calculate Fisher’s Price Index for the year 2011.
(ii) If Fisher’s Price Index for the year 2012 is 110.7, calculate the Paasche’s Price Index for the year 2012, taking 2010 as base year.

Q.6 In an effort to reduce crimes, the Superintendent Police of Far Town has requested the Inspector General to increase police strength in his town. He has gathered information from other towns of the city and submitted the following details to support his request:

<table>
<thead>
<tr>
<th>Towns</th>
<th>Bee</th>
<th>Cee</th>
<th>Dee</th>
<th>Gec</th>
<th>Jay</th>
<th>Kay</th>
<th>Pee</th>
<th>Tee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Police Strength</td>
<td>150</td>
<td>170</td>
<td>250</td>
<td>270</td>
<td>170</td>
<td>120</td>
<td>110</td>
<td>220</td>
</tr>
<tr>
<td>No. of Crimes per month</td>
<td>170</td>
<td>110</td>
<td>50</td>
<td>40</td>
<td>90</td>
<td>210</td>
<td>188</td>
<td>60</td>
</tr>
</tbody>
</table>

(a) Determine the regression equation and interpret your result. (Assuming that ratio of police strength to total number of people is same in all towns)
(b) Determine the coefficient of correlation and determination and interpret your results.
(c) Using the above regression equation, determine whether police of Jay town is more efficient than police of Pee town.

Q.7 (a) A multiple choice examination consists of ten questions and each question is followed by four choices. A student will pass the exam if he answers five questions correctly. Assuming that a student knows two correct answers and chooses the remaining answers at random, what is the probability that he will pass the test?

(b) In a population, which is normally distributed, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the population.

Q.8 (a) A Production House has carried out a survey to assess the popularity of one of its programs. A random sample of 2,000 people was selected and they were asked to give their views. The results are as follows:

<table>
<thead>
<tr>
<th>Opinion</th>
<th>Number of persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like the program</td>
<td>1,040</td>
</tr>
<tr>
<td>Do not like the program</td>
<td>650</td>
</tr>
<tr>
<td>Do not know</td>
<td>310</td>
</tr>
</tbody>
</table>

(i) Construct a 97% confidence interval for the proportion of viewers who like the program.
(ii) Based on the interval constructed by you in part (i), can you say with 97% confidence that majority of the viewers like the program. Would your decision change if the required confidence level is 92%?

(b) A television (TV) manufacturer claims that the mean life of the picture tubes of its classic TV brand is 8,000 hours. A random sample of 18 picture tubes showed a mean life of 7,850 hours with a standard deviation of 150 hours. Test the manufacturer’s claim using a significance level of 0.01.

(THE END)
Q.1 (a) Chemicals A & B are formed by mixing x, y and z in the ratio of 8:3:4 and 4:3:8 respectively. The cost per litre of x, y and z is Rs. 300, Rs. 240 and Rs. 180 respectively. Chemical C is produced by mixing Chemicals A & B in a certain ratio. If total cost of producing 1,000 litres of Chemical C is Rs. 252,000 find the ratio of Chemicals A & B in Chemical C. (06)

(b) Nasim Electronics Limited has designed a household appliance. It has estimated that 1000 units would be produced during the first month. Thereafter, the production would increase at 5% per month for the next 24 months and then start declining by 12% per month till it reaches 250 units per month after which the production would be discontinued.

Compute the total number of units that the company would produce. (08)

Q.2 (a) In October 2011 Aslam had deposited an amount in an investment scheme at 12% interest compounded quarterly, with the objective of receiving Rs. 1,000,000 at the end of the 5th year i.e. October 2016. He has been informed that with effect from 1 October 2013 the interest rate on the scheme for the remaining period would be reduced to 10%.

Determine the amount that Aslam should deposit on 1 October 2013 to have the required amount in October 2016. (06)

(b) Kiran deposited Rs. 5,000 per month (first day of the month) in a saving account in the year 2011 and Rs. 7,500 per month in the year 2012. Find the total amount saved by her at the end of year 2012 if she earned interest @ 8% compounded monthly. (05)

(c) Find the face value of a bill which was discounted by a bank for Rs. 95,000 five months before maturity, at a discount rate of 12% (02)

Q.3 (a) If \( y = e^{2x^2} \ln x^3 \)

Prove that \( \frac{dy}{dx} = \frac{1}{x} \left( 4x^2y + 3e^{2x^2} \right) \) (04)

(b) The average cost and total revenue functions for “q” number of memory cards are as follows:

\[
AC = q + 20 + \frac{1500}{q} \\
TR = 860q - 3q^2
\]

Calculate the maximum profit that can be earned and the price at which the profit would maximise. (07)

Q.4 (a) Solve the following system of equation by using Cramer’s Rule:

\[
x - y - 5z = 4 \\
2x - 3y - 3z = 3 \\
3x - 2y - 7z = 2
\]

(08)

Continued on next page....
(b) Sketch the feasible region for the following set of constraints:
\[ \begin{align*}
x & \leq 5 \\
y & \geq 2 \\
x + y & \leq 8
\end{align*} \] (04)

Q.5  
(a) Draw a histogram and a frequency polygon for the following frequency distribution relating to the ages of the students in a school:

<table>
<thead>
<tr>
<th>Students’ ages (years)</th>
<th>6–7</th>
<th>8–9</th>
<th>10–12</th>
<th>13–15</th>
<th>16–19</th>
</tr>
</thead>
</table>
| No. of students        | 6   | 8   | 21     | 15     | 16    | (05)

(b) Calculate the median, mean deviation from median and coefficient of mean deviation from median of the following frequency distribution:

<table>
<thead>
<tr>
<th>Class intervals</th>
<th>0–9</th>
<th>10–19</th>
<th>20–29</th>
<th>30–39</th>
<th>40–49</th>
</tr>
</thead>
</table>
| Frequency       | 10  | 14    | 24    | 15    | 09    | (07)

Q.6  
(a) The marks obtained by eight students in a competency test are as follows:

<table>
<thead>
<tr>
<th>Student ID</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounting</td>
<td>78</td>
<td>80</td>
<td>80</td>
<td>85</td>
<td>65</td>
<td>60</td>
<td>69</td>
<td>72</td>
</tr>
<tr>
<td>Mathematics</td>
<td>90</td>
<td>87</td>
<td>85</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>70</td>
<td>73</td>
</tr>
</tbody>
</table>

Find out the Spearman’s coefficient of rank correlation for the above marks and interpret the result. (07)

(b) Find out the regression coefficient of y on x, if regression coefficient of x on y is 0.25 and the coefficient of correlation between x and y is 0.6. (02)

Q.7  
(a) Inaam is going to play a series of 3 tennis matches with Misbah. Assuming that chances of winning and losing are equal, find the following probabilities for Inaam:

(i) Two wins and one loss  
(ii) At least two wins  
(iii) No loss  
(iv) At most two losses (05)

(b) APZ Limited produces a component having a diameter of 3.0 cm. A customer has ordered 100,000 units and has indicated that he would be willing to accept a variation of up to 0.01 cm. The diameter of the component has a normal distribution with mean of 3.0 cm and standard deviation = 0.005 cm.

Estimate the number of components that the customer would reject. (04)

(c) Small Insurance Company receives an average of 8 insurance claims daily during the month of Ramadan every year. Using Poisson distribution, find the probability that on a certain day in Ramadan the company will receive:

(i) no insurance claim.  
(ii) less than four insurance claims.  
(iii) at least two insurance claims. (06)

Q.8  
(a) Find the probability of getting a sample mean within the range of 1.3% of the population mean, if a sample of 36 packages is drawn at random from a population having standard deviation of 4% of the population mean. (06)

(b) Heights in inches, of ten individuals chosen at random from a normal population, are as follows:

63, 63, 66, 67, 68, 69, 70, 70, 71, 71

In the light of the above data, discuss the suggestion that the mean height of the population at significance level of \( \alpha = 0.05 \) is 66 inches. (08)

(THE END)
Q.1  
(a) The ratio of boys to girls in a school is 7:6. If the number of boys and girls in the school increases by 10% and 15% respectively in one year, the number of boys will exceed the number of girls by 120. What is the total number of students in the school?  
(b) Factorize the following expression to its simplest form: 
\[x^4 - 22x^2y^2 + 9y^4\]  
(c) The equation: 
\[y = -50,000x + 1,000,000,\] represents the value of a car after \(x\) years.  
(i) Find the slope of the line and interpret its meaning.  
(ii) Find the \(y\)-intercept and explain what it represents.  

Q.2  
(a) Saad is to retire after 10 years. He wants to have sufficient money to be able to withdraw Rs. 6 million, 12 years from now, for his son’s higher education and Rs. 100,000 per month for household expenses for a period of 15 years after his retirement.  
Saad intends to invest all his money in a bank which offers a guaranteed return of 9% compounded monthly. His present savings amount to Rs. 2 million and he expects to receive Rs. 8 million when he retires after 10 years. He also plans to make monthly contributions in the above account until the date of his retirement.  
Determine the amount that Saad should deposit in the above bank account each month in order to be able to withdraw the desired amounts.  
(b) Rashid wants to invest Rs. 1,500,000 for one year. Bank A has offered simple interest @ 10% per annum, whereas Bank B has offered interest @ 8% per annum compounded half yearly.  
Rashid believes that investment in Bank A is risky and therefore he prefers to invest in Bank B. However, as he wants to earn interest of at least Rs. 140,000, he would also have to invest some of the amount in Bank A.  
Determine the minimum amount that he will have to invest in Bank A to achieve the above objective.  

Q.3  
(a) If \(y = \ln(2e^{5x} + 5e^{3x} - 16)\) find:  
(i) \(\frac{dy}{dx}\) and  
(ii) the exact value of the co-ordinates of the point where \(\frac{dy}{dx} = 5\).  
(b) Find maxima, minima and point of inflexion (if any) of the following function:  
\[y = 3x^4 - 16x^3 + 24x^2 - 5\]
Q.4  (a) A departmental store has three outlets. During a certain week, it sold 1 unit of product x, 6 units of product y and 1 unit of product z at the first outlet; 5 units of product x, 6 units of product y and 4 units of product z at the second outlet; and 3 units of product x, 1 unit of product y and 8 units of product z at the third outlet. Total profit generated by each outlet from these three products was Rs. 2,500, Rs. 5,200 and Rs. 2,900 respectively. Calculate the profit per unit of product x, y and z using Cramer’s Rule.  (08)

(b) A company manufactures two types of stereo systems M and N. Each unit of system M requires 3 worker-hours for process A and 2 worker-hours for process B while each unit of system N requires 4 worker-hours for process A and 6 worker-hours for process B. Available skilled labour allows for a maximum of 480 worker-hours per week for process A and 540 worker-hours per week for process B. How many units of each type of stereo systems may be produced to maximise the profit, if the profit on each unit of M and N is Rs. 2,000 and Rs. 5,000 respectively?  (08)

Q.5  (a) The following data depicts ages (in years) of persons applying for new membership in a club:


<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>18</th>
<th>15</th>
<th>47</th>
<th>23</th>
<th>26</th>
<th>41</th>
<th>19</th>
<th>26</th>
<th>28</th>
<th>29</th>
<th>46</th>
<th>49</th>
<th>23</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>43</td>
<td>34</td>
<td>18</td>
<td>24</td>
<td>29</td>
<td>39</td>
<td>21</td>
<td>35</td>
<td>28</td>
<td>34</td>
<td>22</td>
<td>20</td>
<td>44</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

(i) On the basis of the above data prepare:
- group frequency distribution and cumulative frequency distribution using seven class intervals.
- cumulative frequency polygon.  (04)

(ii) Calculate the median and mode for the above frequency distribution.  (04)

(b) A consignment of 12 refurbished laptops contains 3 defective units. If 4 laptops are randomly chosen for inspection, what is the probability that at least 2 of them will be defective?  (03)

Q.6 Below is the list of averages of batsmen (rounded to whole number) in ODI and Test matches.

<table>
<thead>
<tr>
<th>Batsman</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>ODI Average</td>
<td>33</td>
<td>42</td>
<td>31</td>
<td>46</td>
<td>36</td>
<td>35</td>
<td>24</td>
<td>39</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>Test Average</td>
<td>44</td>
<td>50</td>
<td>38</td>
<td>42</td>
<td>31</td>
<td>44</td>
<td>31</td>
<td>35</td>
<td>41</td>
<td>49</td>
</tr>
</tbody>
</table>

(a) Draw a Scatter diagram for the above data.  (02)

(b) Find the coefficient of correlation for averages in ODI and Test matches and interpret your result.  (06)

(c) Calculate the probable error of co-efficient of correlation and interpret your result.  (03)

Q.7  (a) Research shows that an outbreak of new virus is causing fatalities in 3 out of every 10 patients affected by the virus. If 12 patients are admitted in a hospital affected with the virus, calculate the probability that:

(i) All patients will survive.
(ii) At least ten patients will survive.
(iii) At most ten patients will survive.  (08)
(b) The life of a light bulb is normally distributed with standard deviation of 100 hours. The probability that the life of a bulb selected at random would exceed 3,200 hours is 0.0228.

What is the probability that the life of a light bulb selected at random would **not** be less than 2,800 hours? 

Q.8  
(a) A company has purchased a new machinery. The machine was tested for 40 days and its mean production was found to be 985 units per day with standard deviation of 30 units.

(i) Construct a 95% confidence interval for the mean production of the population.
(ii) Determine the size of the sample which would be needed in order to assert with 99% confidence that the error in the determination of production capacity does not exceed 10 units.

(b) A university has two campuses. Two batches of 60 students from each campus were surveyed. Batch A had 15 counts of 85% marks or above, whereas Batch B had 10 counts of 85% or above.

(i) Construct a point estimate for difference between the counts of Batch A and B.
(ii) Calculate the standard deviation of the difference of the above population proportion.

(THE END)